

Un imageur compressé utilisant les milieux multiplement diffusants

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Un imageur compressé utilisant les milieux multiplement diffusants ou “La vérité si je m’embrouille”

Laurent Daudet

et I. Carron, G. Chardon, S. Gigan, O. Katz,
G. Lerosey, A. Liutkus, D. Martina, S. Popoff



Notion de mesure

Mesurer : Déterminer une quantité ou une grandeur en la comparant à une quantité ou une grandeur de même nature prise comme référence.

Dictionnaire Académie (9è Ed)

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Notion relative à une référence,
quantitative : on obtient un nombre

Notion de mesure en physique

Tous les objets physiques se laissent-ils décrire par des nombres ?

Décrire un objet: posséder un modèle (a priori) puis mesurer le ou les paramètre(s) de ce modèle

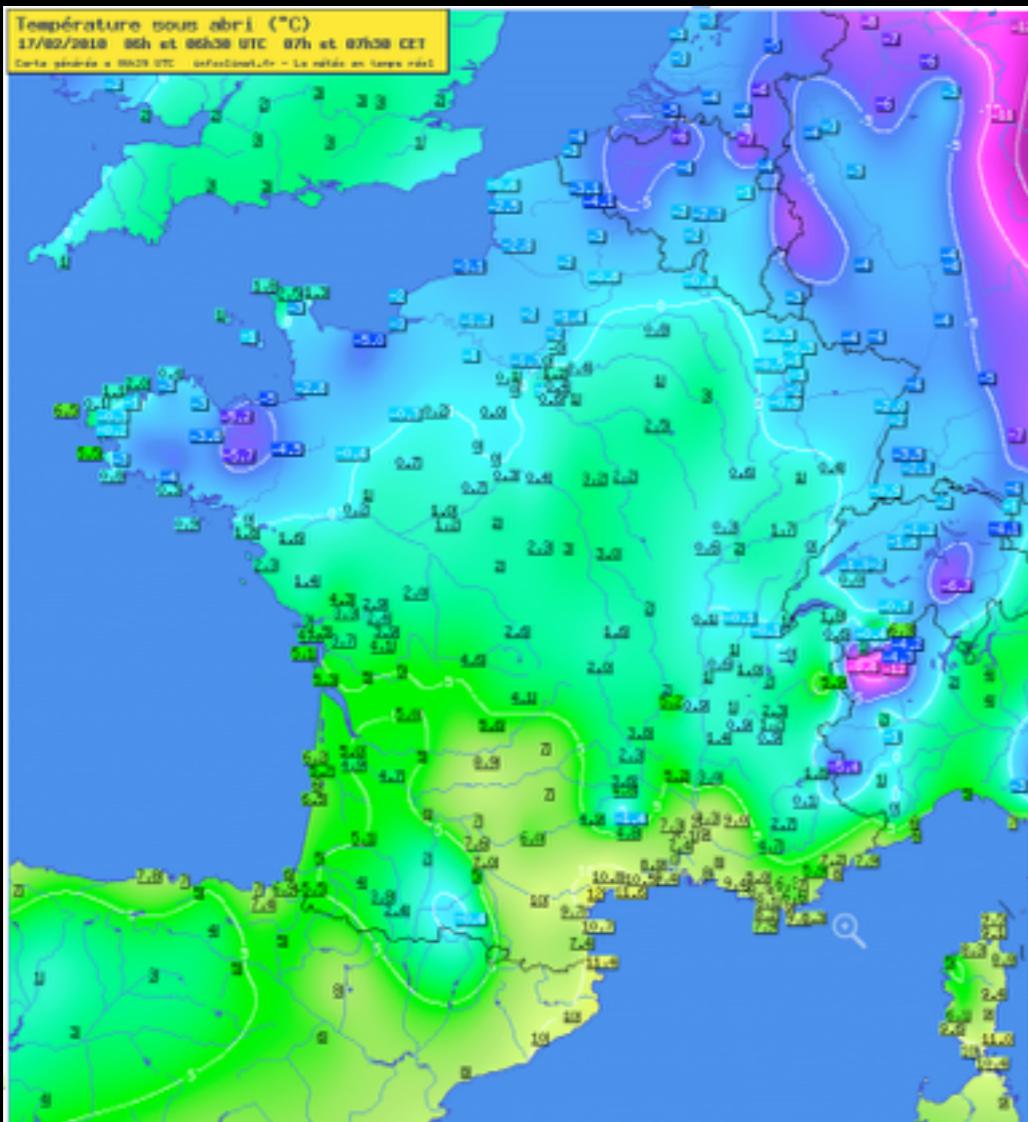
Plus on a d'a priori, moins on a besoin de mesurer.

Exemple : les moutons de Dirac

Echantillonnage : pour mesurer le continu

Problème des objets dépendant de paramètres continus :
exemple temps, espace

Ex température = $T(\text{latitude}, \text{longitude}, \text{temps})$



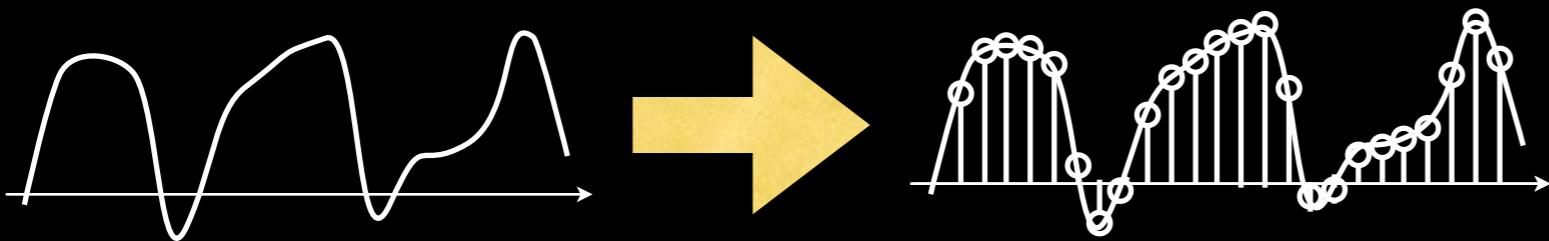
Echantillonnage : pour mesurer le continu

Comment connaitre la T partout en France alors que l'on n'est pas capable de la mesurer partout ?

Problème théorique d'*échantillonnage*
(notion utile aussi en statistique : connaître le tout en ne mesurant qu'une partie)

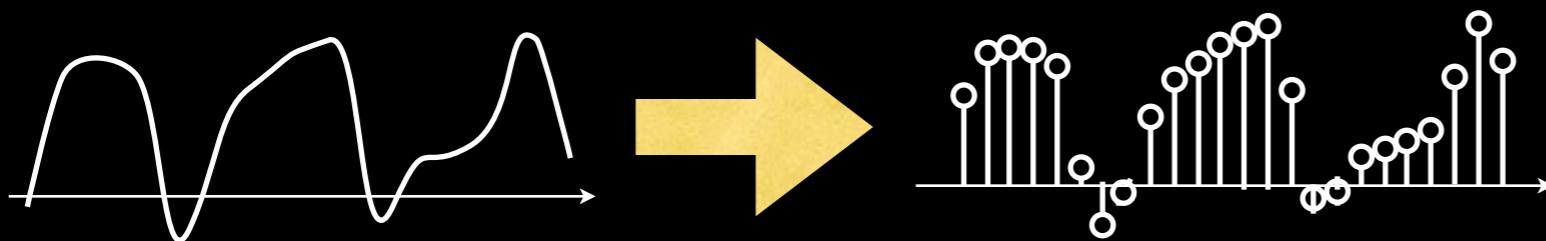
Problème concret de *numérisation* :
on peut pas manipuler une infinité de nombres sur un ordinateur (par ex pour images, sons, etc ...)

Echantillonnage version 40's - 50's



Si les variations ne sont pas trop rapides, la courbe est entièrement déterminée par des échantillons régulièrement espacés

Echantillonnage version 40's - 50's



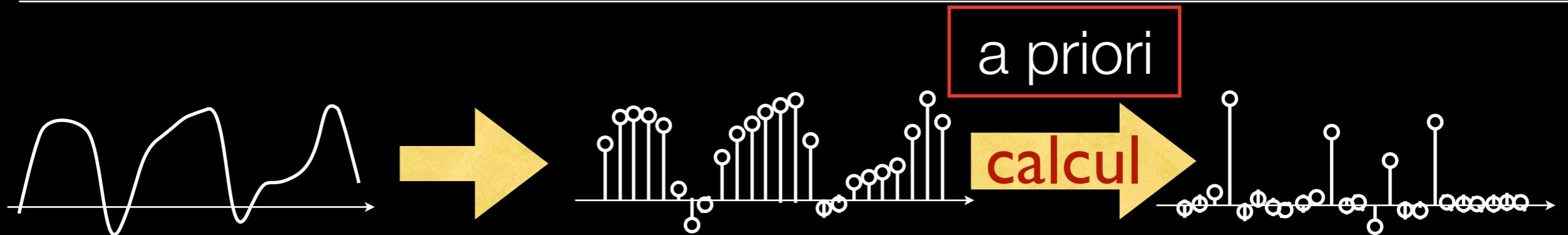
Si les variations ne sont pas trop rapides, la courbe est entièrement déterminée par des échantillons régulièrement espacés

Lien entre distance entre échantillons et variations du signal : $f_e > 2 f_{\max}$

Shannon-Nyquist-Whittaker-Kotelnikov-Raabe

Reconstruction *linéaire* à partir des échantillons

Traitement des signaux version 80's - 90's



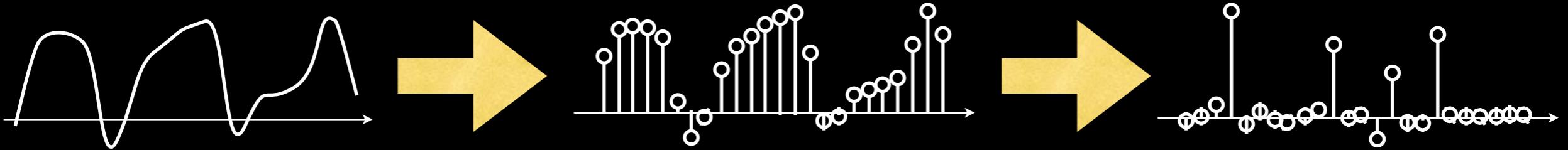
Par changement de base (ou dans une base redondante), la plupart des signaux naturels sont « parcimonieux » :

peu de gros coefficients, beaucoup de très petits

Permet le débruitage, le codage, etc ...

Nécessite un *a priori* sur quelle est la base appropriée

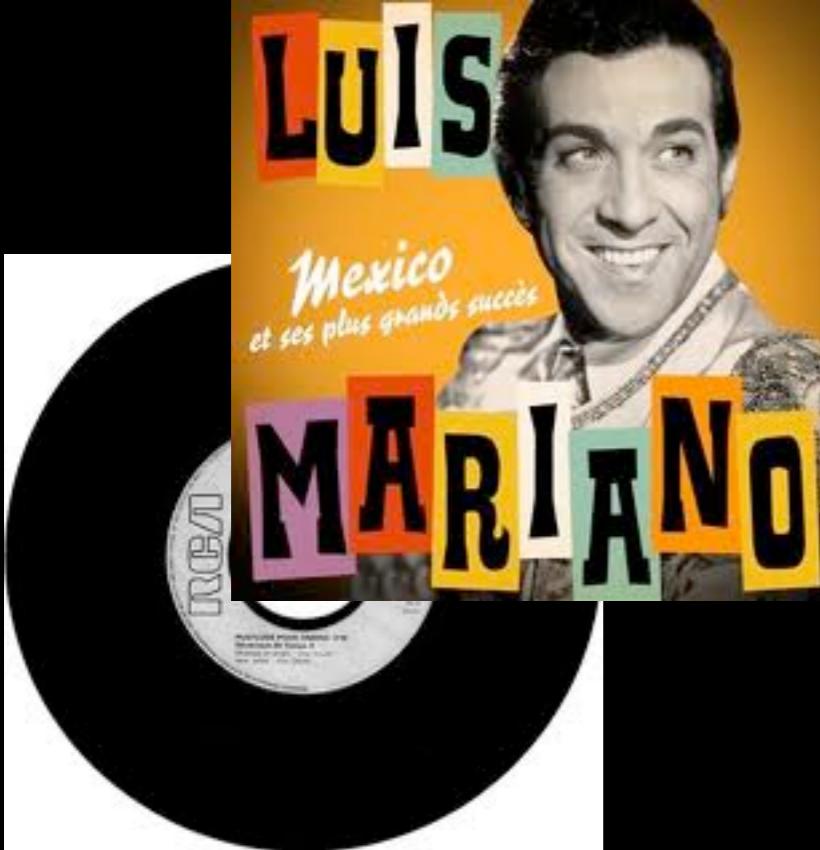
Exemple : la musique



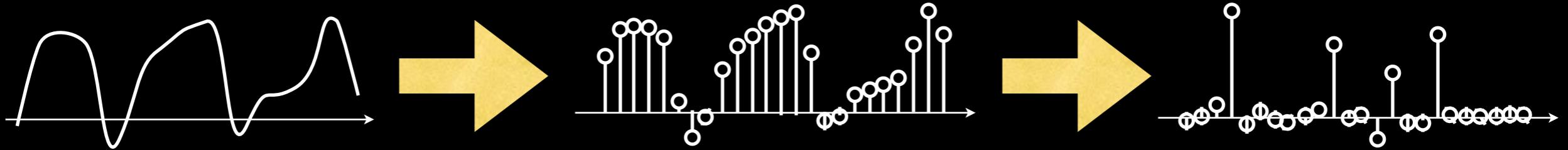
$p(t)$
analogique

0101001101001010101110011100
0101010101011100111001001010
1110000101010001010100111000
00011001000101011100111010
1100111011001010111001100100
1100111011001010111001100100
1100111011001010111001100100

01010011010010
10101110011100
01010101010111



Exemple : les images



I(x,y)

argentique



numérique
RAW



0101001101001010101110011100
0101010101011100111001001010
1110000101010001010100111000
0001100100010101110011101010
1100111011001010111001100100
1100111011001010111001100100
1100111011001010111001100100

01010011010010
10101110011100
01010101010111

numérique
JPG



Les années 2000-2010: Echantillonnage compressé



Candès, Donoho (2006) : OUI si le signal est parcimonieux
dans une base B1 ET si on acquiert dans une base B2
incohérente avec B1

Bonne manière d'obtenir l'incohérence : le **hasard** !

Echantillonnage compressé

Exemple : la bataille navale !

Echantillonnage compressé

Exemple : la bataille navale !



Echantillonnage compressé

Exemple : la bataille navale

	1	2	3	4	5	6	7	8	9	10
A		●								
B										✗
C							●			
D									●	
E			✗							
F							✗			
G				●						
H					●			✗		
I									✗	
J										

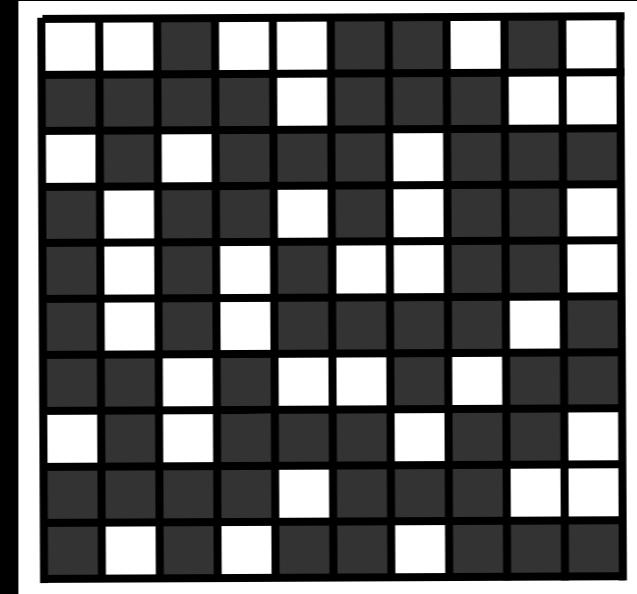
Retrouver la position
des 5 bateaux en
posant le moins de
questions possibles

(ici en moyenne il faut
en 85 questions)

Echantillonnage compressé

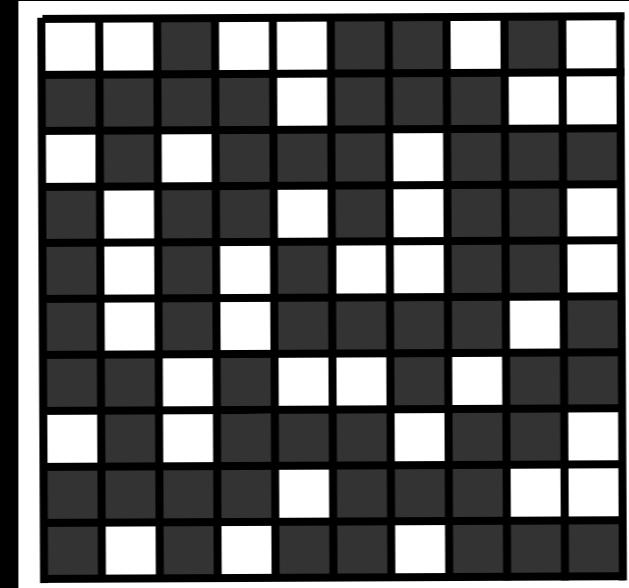
Exemple : la bataille navale

	1	2	3	4	5	6	7	8	9	10
A		●								
B										
C						●				
D							●			
E										
F										
G			●							
H				●						
I										
J										



Echantillonnage compressé

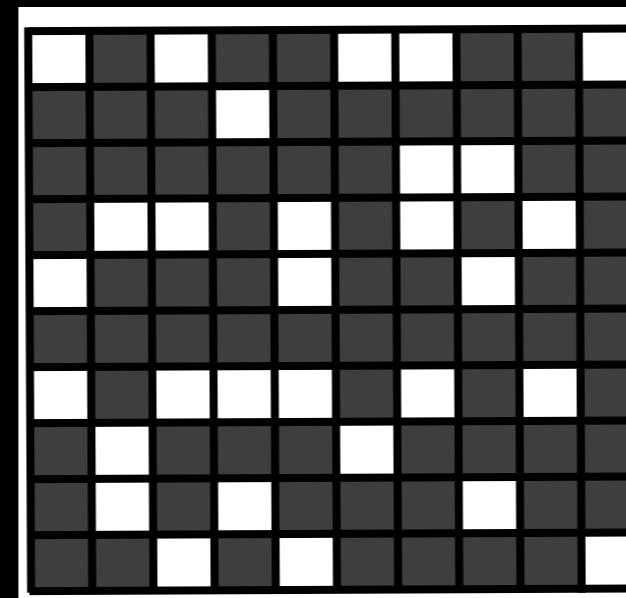
Exemple : la bataille navale



A 10x10 grid puzzle with numbered columns (1-10) and lettered rows (A-J). Columns 1-4 are light blue, and columns 5-10 are dark grey. Red circles are located at the following coordinates: (A, 2), (D, 9), (G, 6), (H, 7), and (I, 1).

Echantillonnage compressé

Exemple : la bataille navale



A 10x10 grid puzzle with numbered columns (1-10) and lettered rows (A-J). The grid contains black and white squares. Red circles are placed at specific intersections: (C, 7), (D, 9), (G, 5), (H, 7), and (I, 1).

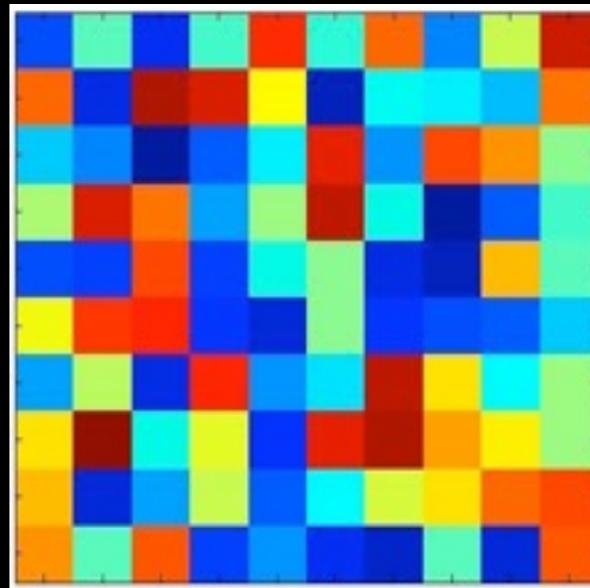
Echantillonnage compressé

Exemple : la bataille navale

\sum

	1	2	3	4	5	6	7	8	9	10
A		●								
B										
C					●					
D						●				
E										
F										
G			●							
H				●						
I										
J										

•*



= m₁

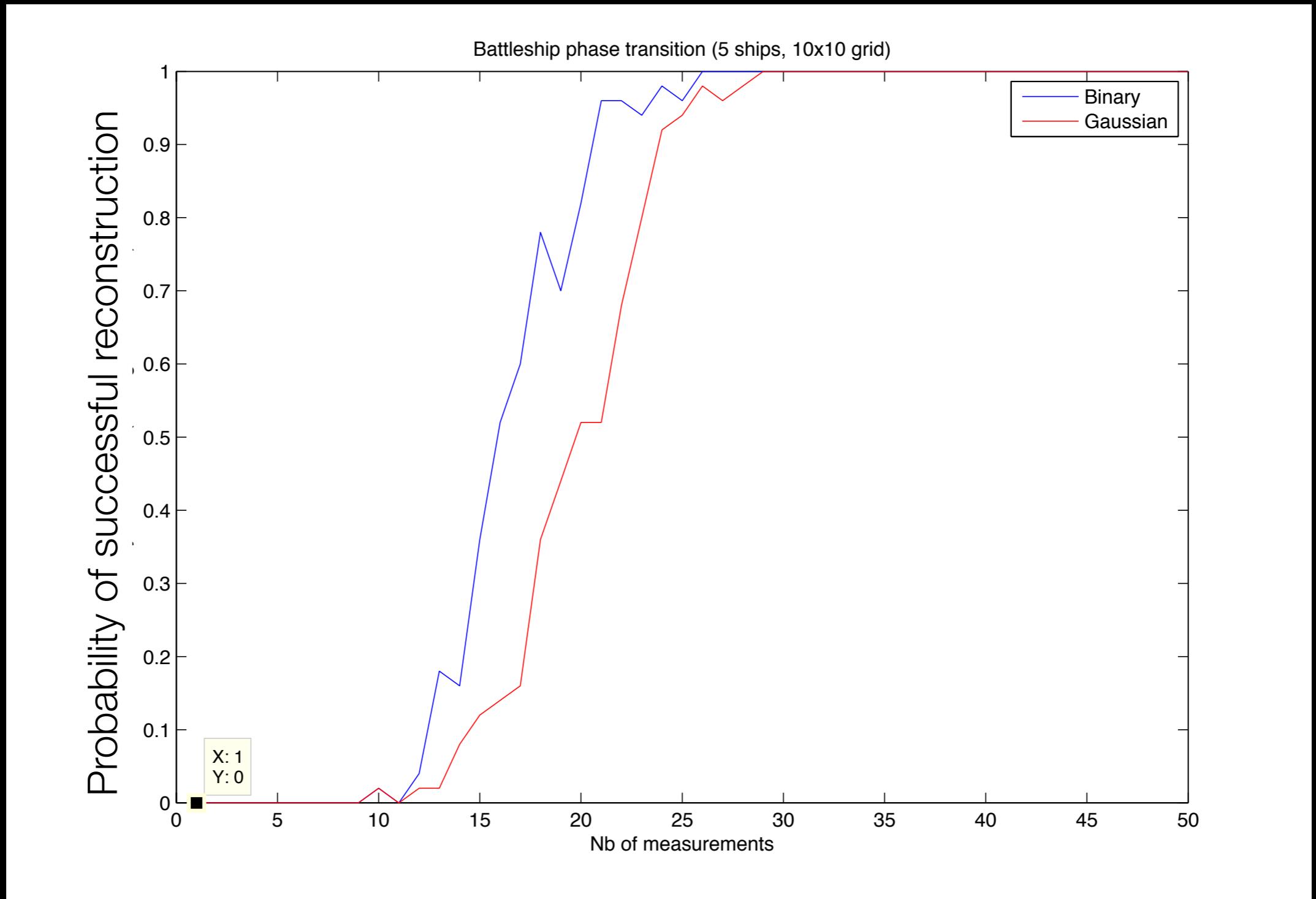
Echantillonnage compressé

Exemple : la bataille navale

$$\sum \cdot * = m_2$$

A diagram illustrating compressed sampling. On the left, a 10x10 grid represents a battle map. The columns are labeled 1 through 10 at the top, and the rows are labeled A through J on the left. Five black dots represent sampled data points at coordinates (A, 2), (C, 7), (D, 9), (G, 4), and (H, 6). To the right of the grid is a symbol consisting of a square followed by a star (*). To the right of that is a 10x10 heatmap with a color gradient from blue to red. The heatmap has numerical labels 1 through 10 along its bottom edge. This heatmap represents the compressed measurement vector m_2 .

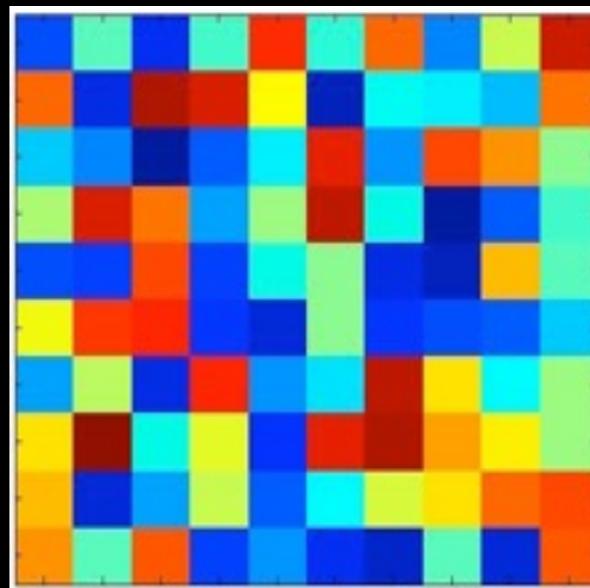
Echantillonnage compressé



Echantillonnage compressé

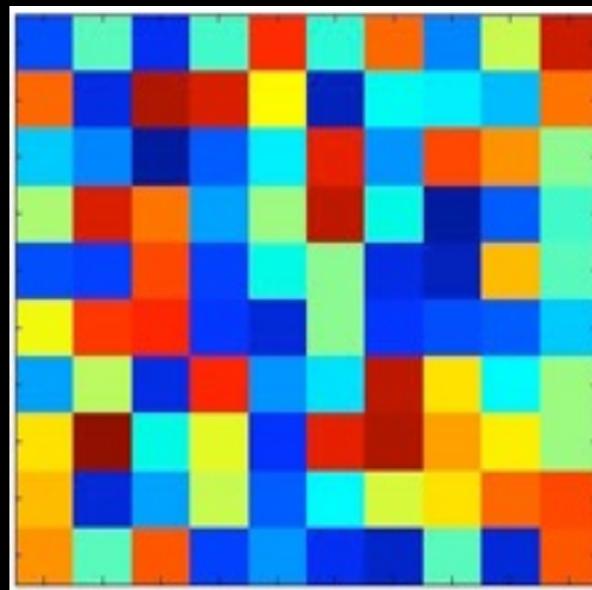
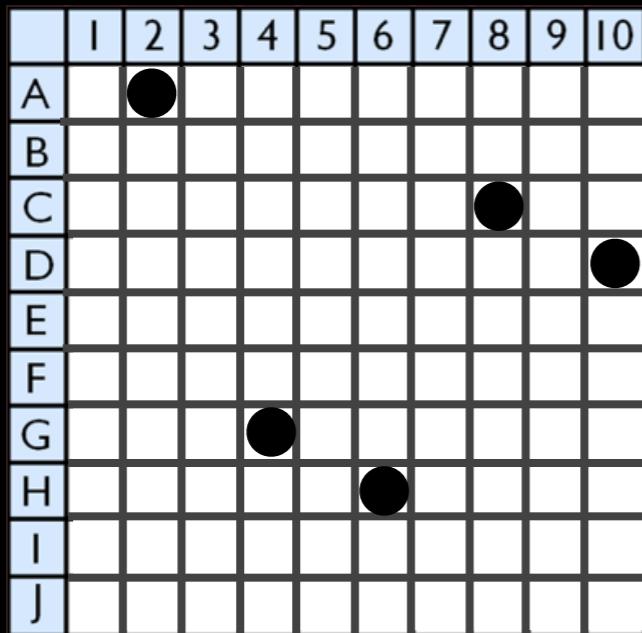
Exemple : la bataille navale

	1	2	3	4	5	6	7	8	9	10
A		●								
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C				●						
D						●				
E										
F										
G			●							
H				●						
I										
J										



Echantillonnage compressé

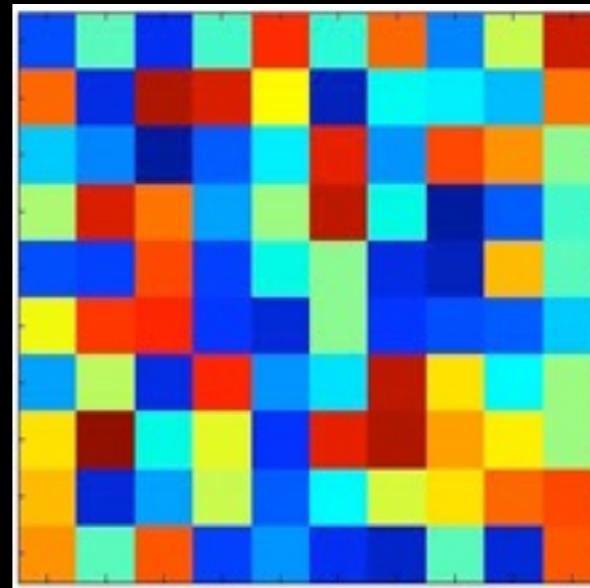
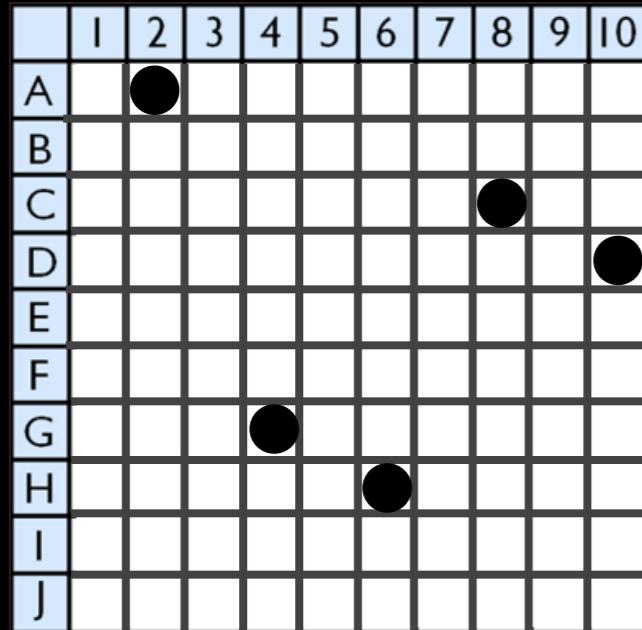
Exemple : la bataille navale



échantillonnage standard, nombre questions \sim nb cases

Echantillonnage compressé

Exemple : la bataille navale

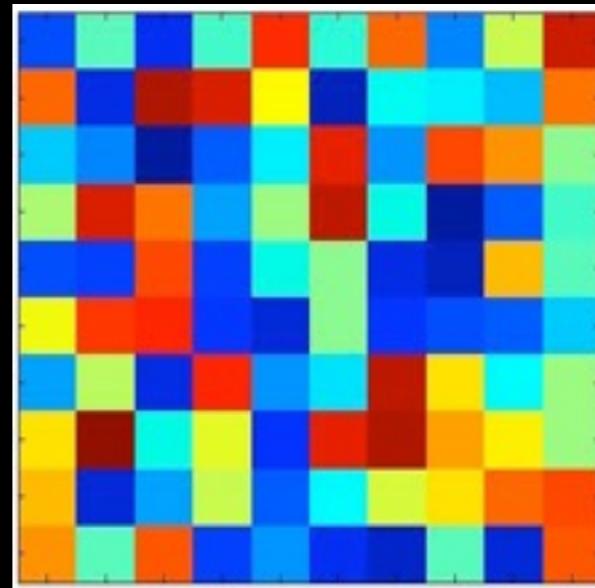
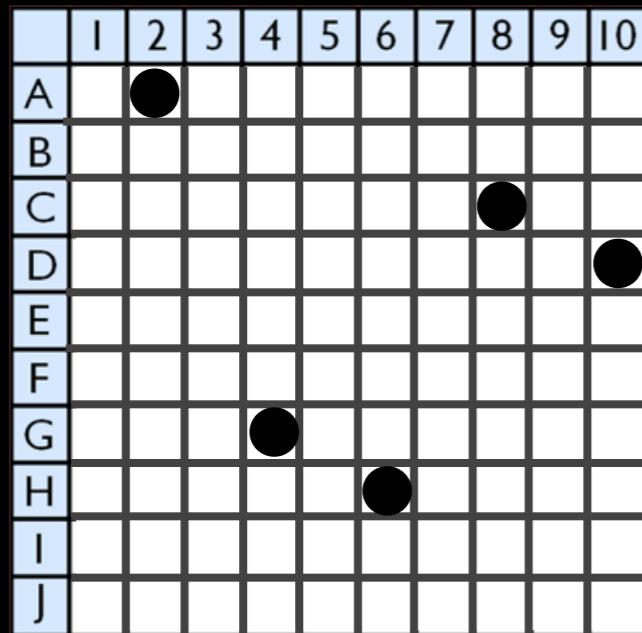


échantillonnage standard, nombre questions \sim nb cases

échantillonnage compressif, nombre bateaux $<<$ nb cases

Echantillonnage compressé

Exemple : la bataille navale



échantillonnage standard, nombre questions \sim nb cases

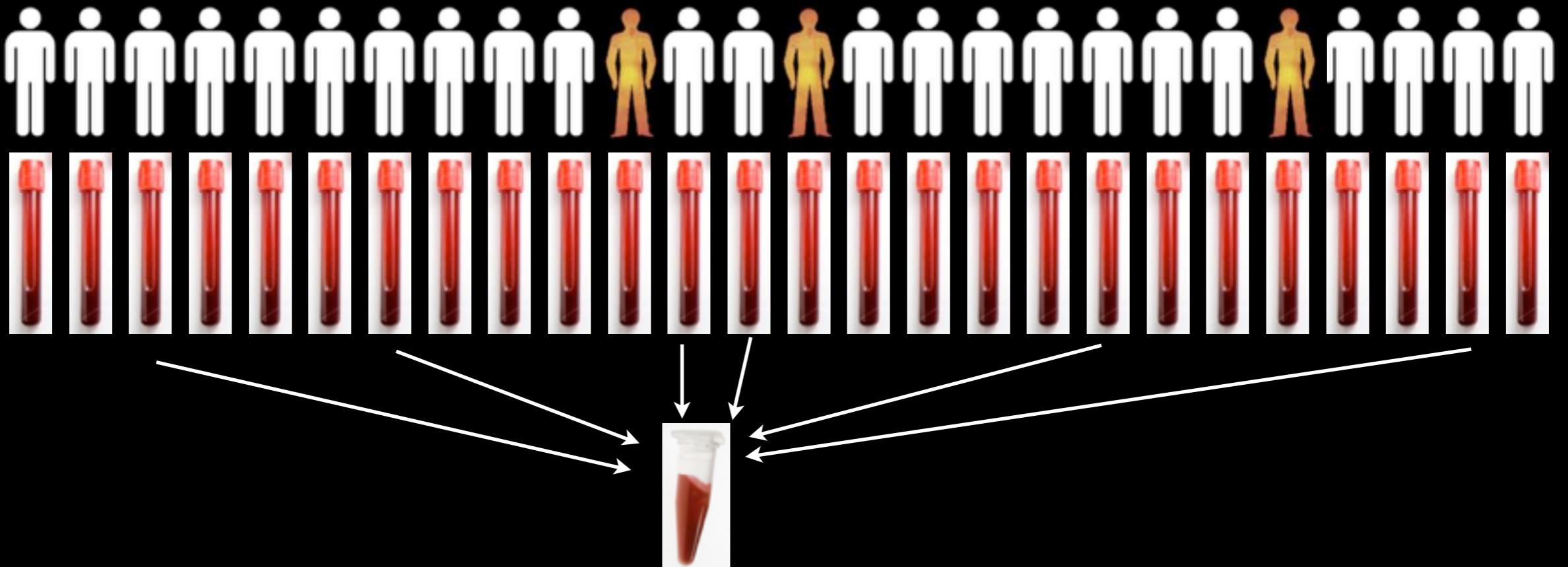
échantillonnage compressif, nombre bateaux \ll nb cases

avec dépendance en $\log(N)$ sur le nombre de cases

Echantillonnage compressé

- **Démocratique** : Chaque mesure donne de l'information sur la totalité du signal (toutes les mesures sont également informatives)
- **Universel** : avec des mesures aléatoires, on mesure de façon incohérente quelle que soit la base de parcimonie.

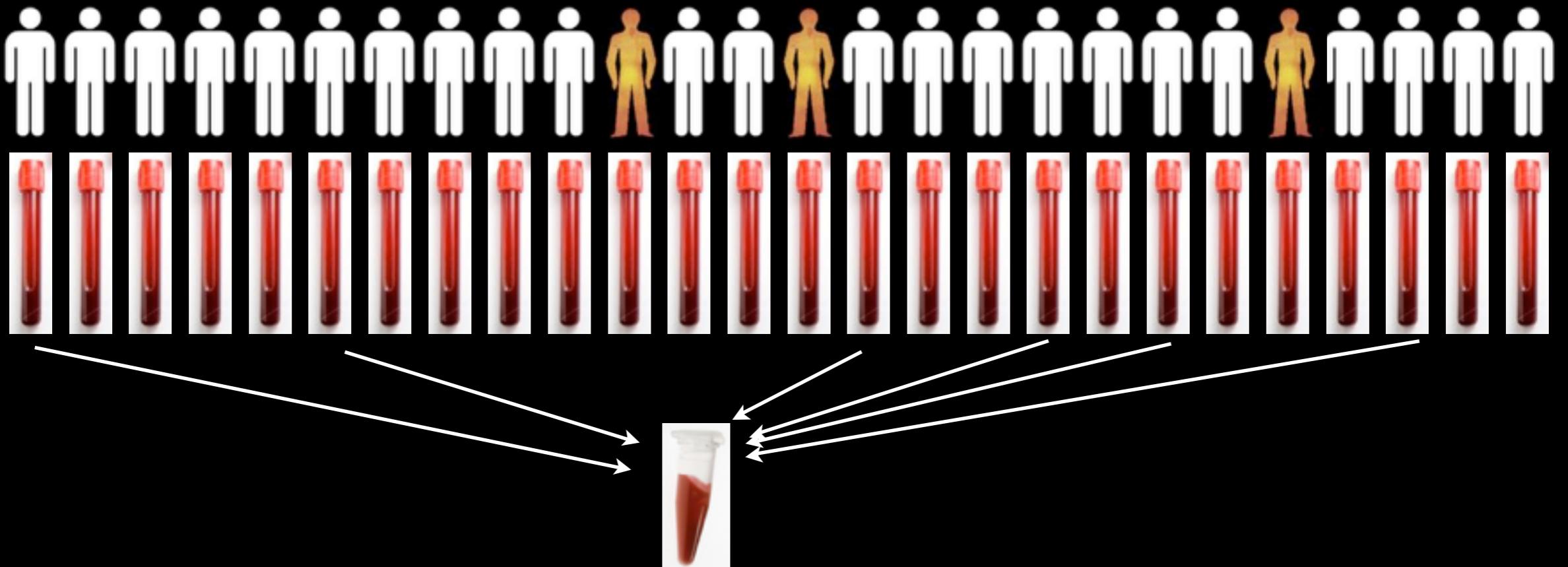
Lien avec le «group testing»



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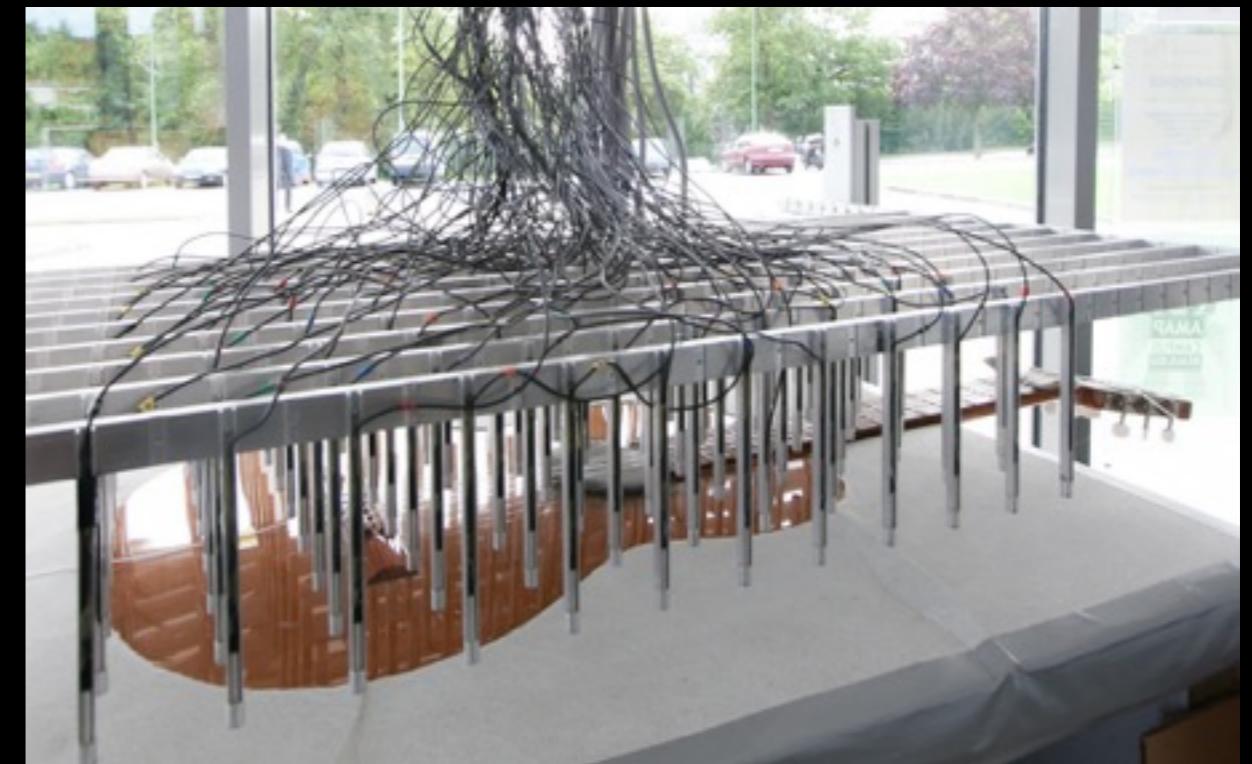
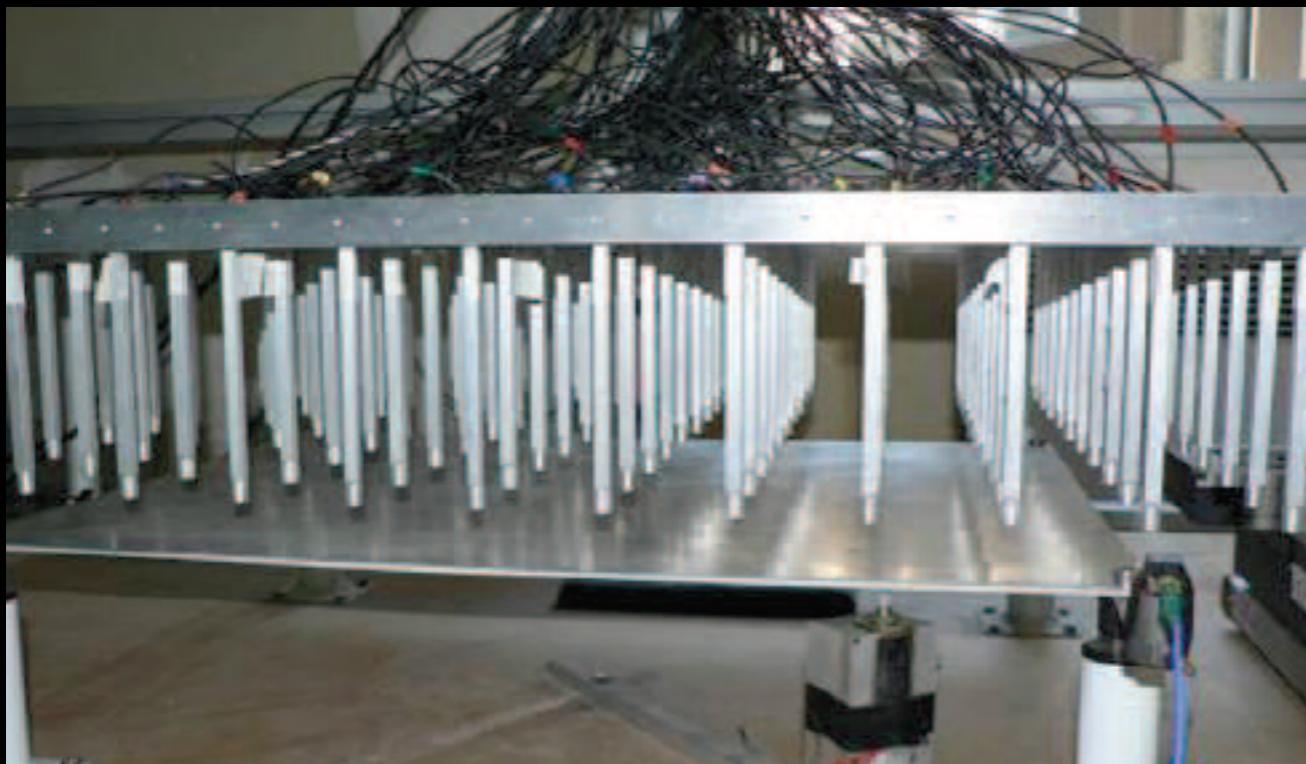
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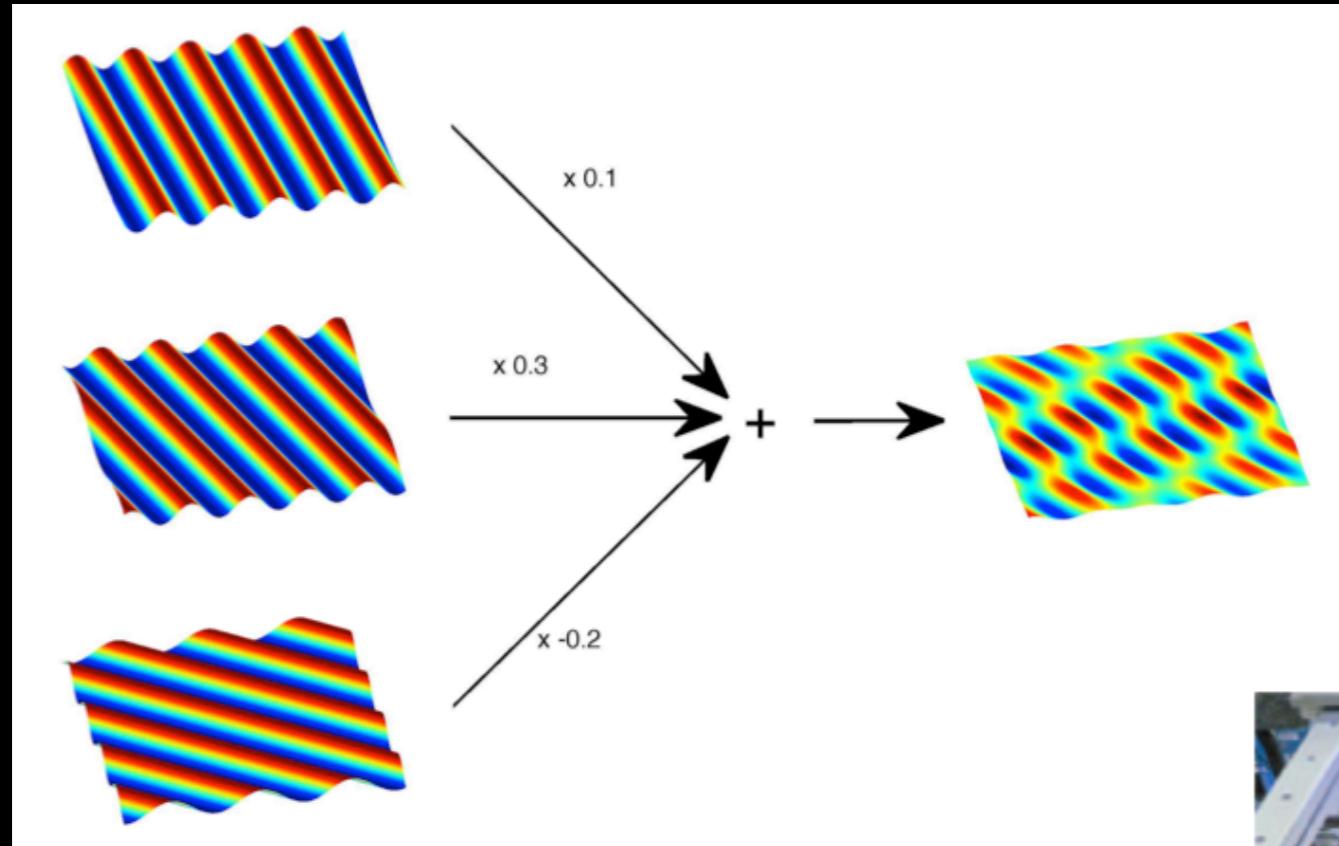
Echantillonnage compressé en acoustique

Holographie Acoustique en Champ Proche

But : comprendre les modes de vibration de plaques vibrantes (tables d'harmonie de guitare par ex).

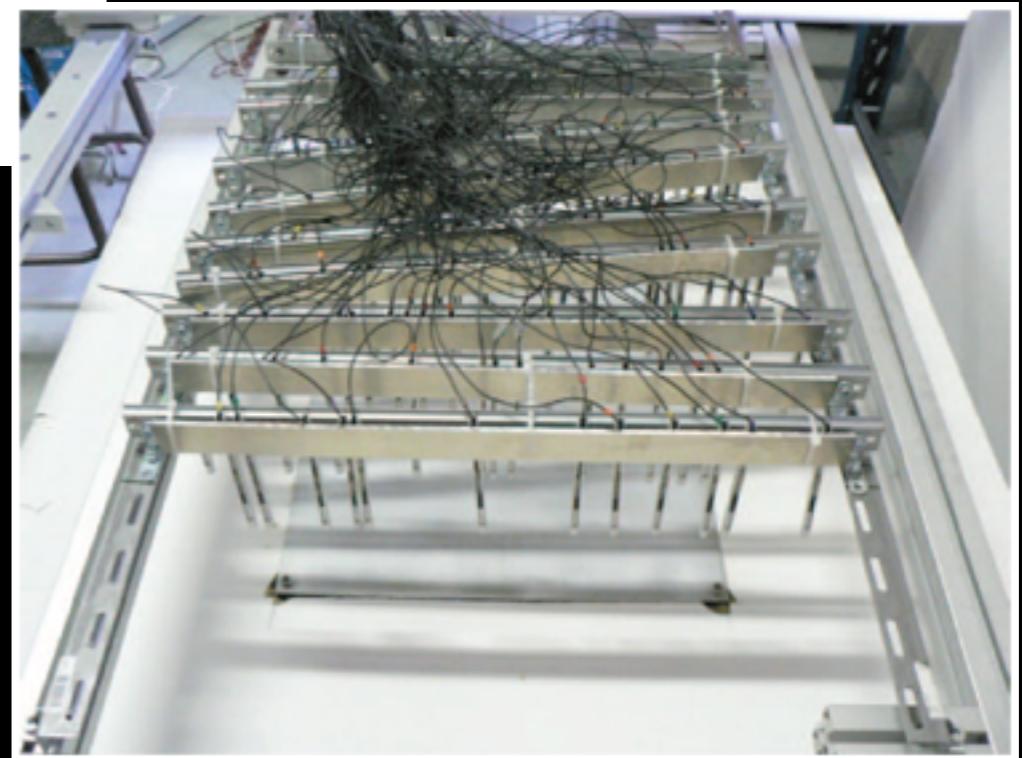


Echantillonnage compressé en acoustique



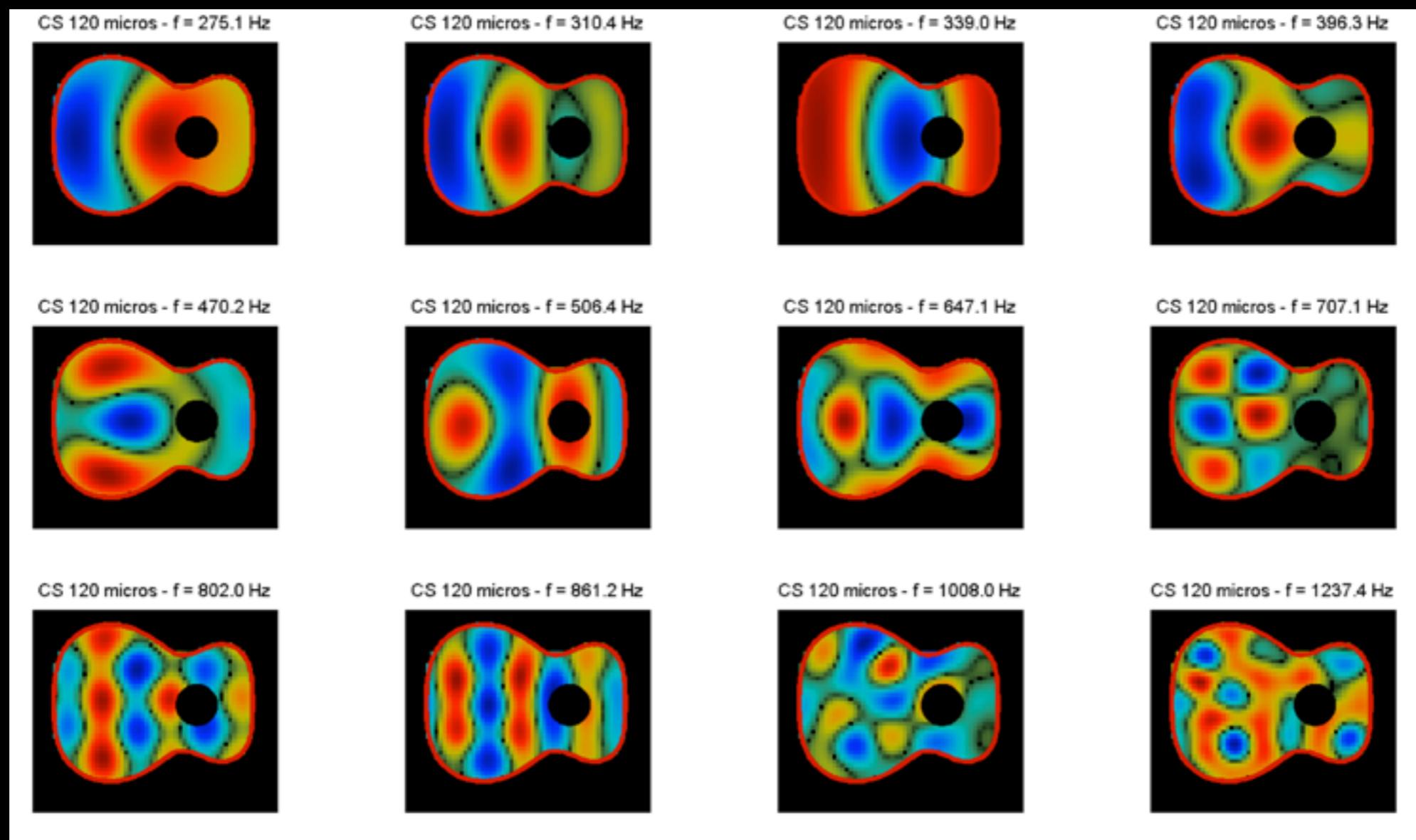
La parcimonie des vibrations dans une base approprié ...

... suggère d'utiliser une antennes à positions de micros aléatoires



Echantillonnage compressé en acoustique

Réduction drastique du nombre de mesures nécessaires !

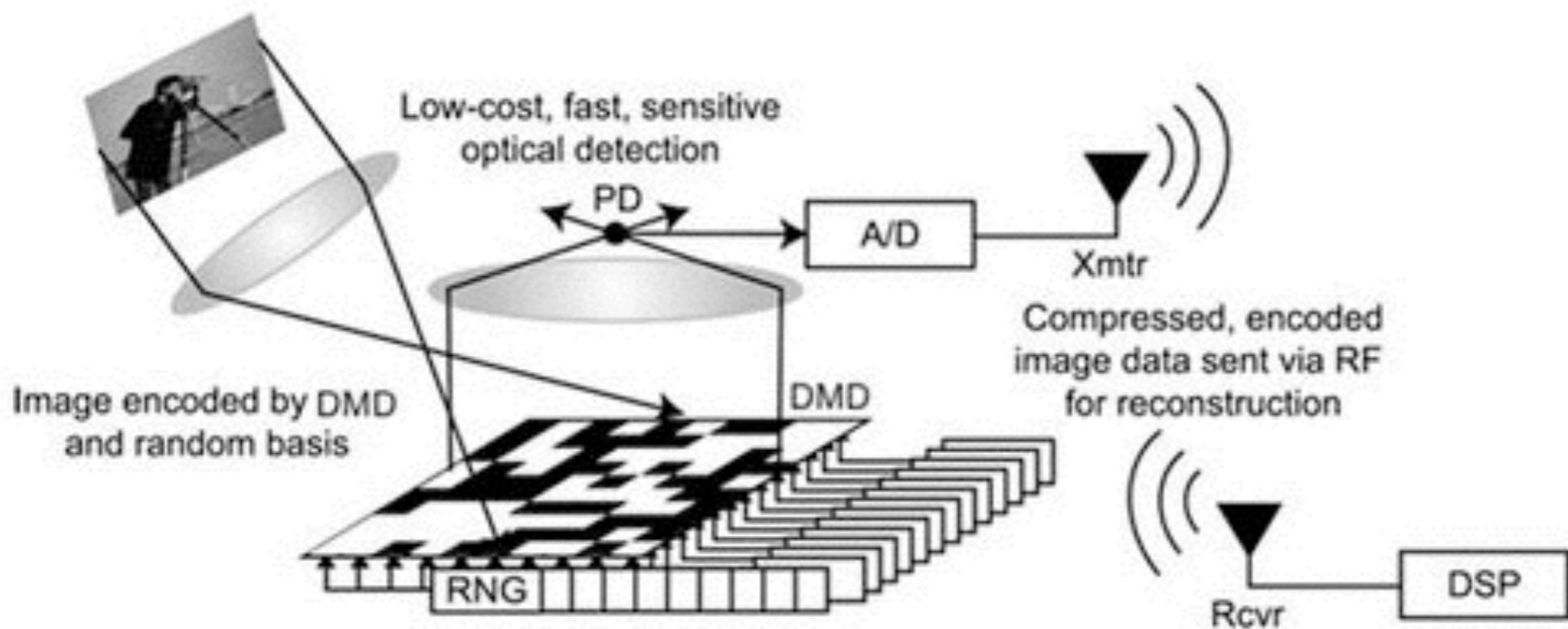


The one-pixel camera

If natural images are sparse, are there better sampling schemes than 20 Mpixel regular sensors as in digital cameras ? (where 99% of images end up as JPEGs)

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(Baraniuk team, Rice Univ.)

Beyond the single-pixel camera

- Information scrambling by randomness is provided by carefully-engineered hardware (as are other similar schemes based on coded aperture)
- Measurements are performed sequentially (slow)

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Can we make it simpler / faster ?

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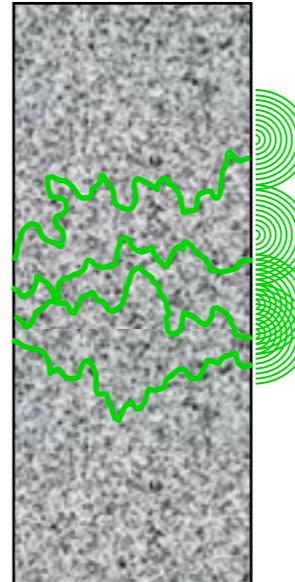
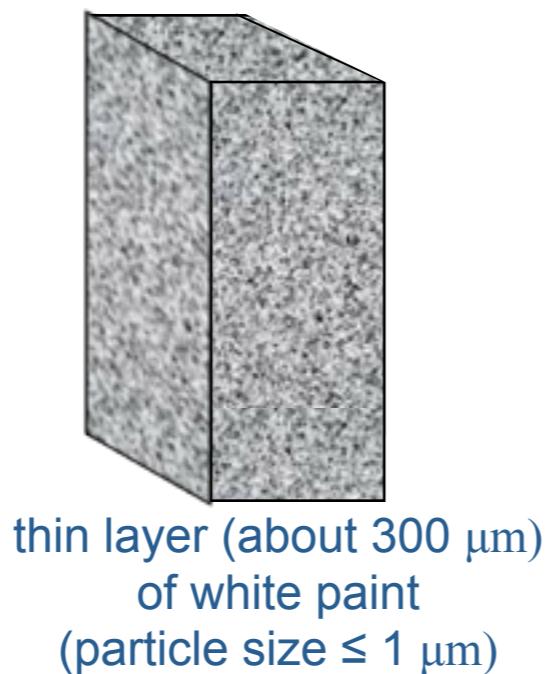
Use randomness provided by Nature
in multiply scattering materials

Scattering : a coherent process

Scattering : a coherent process

Volume scattering:

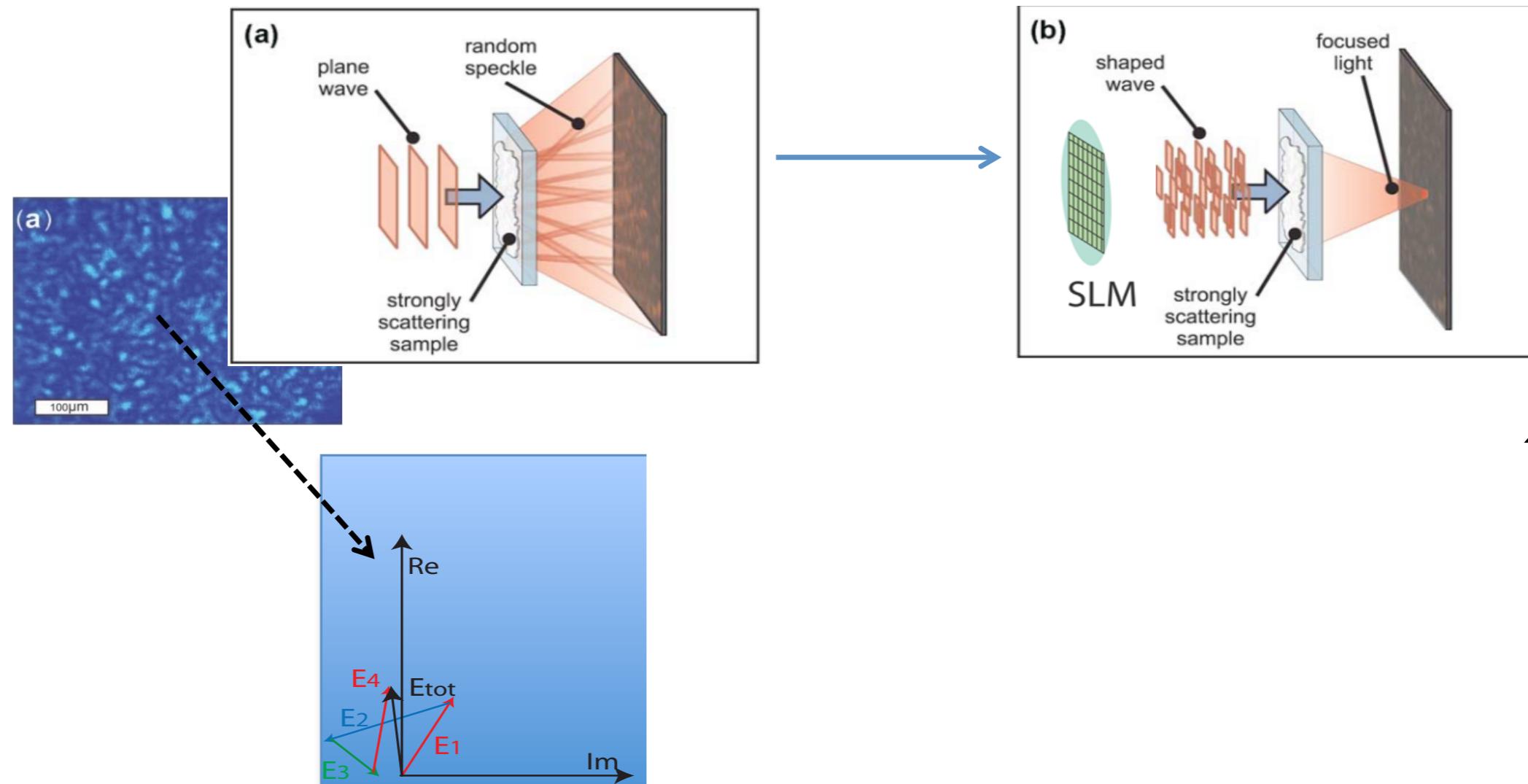
Coherent light
(laser)



Speckle results from multiple interference
between a multiplicity of random paths

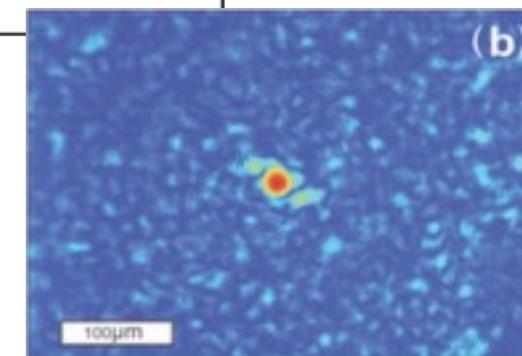
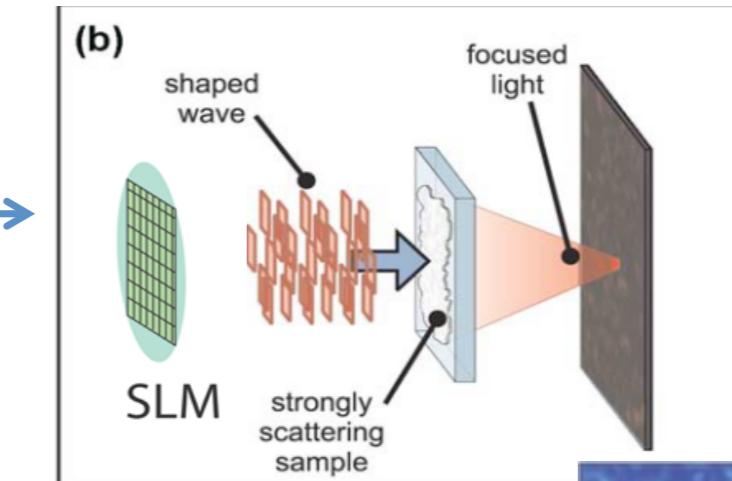
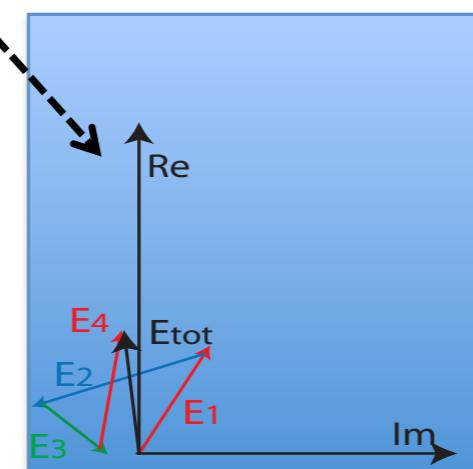
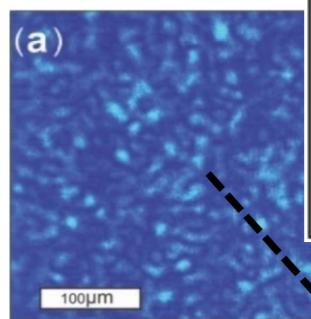
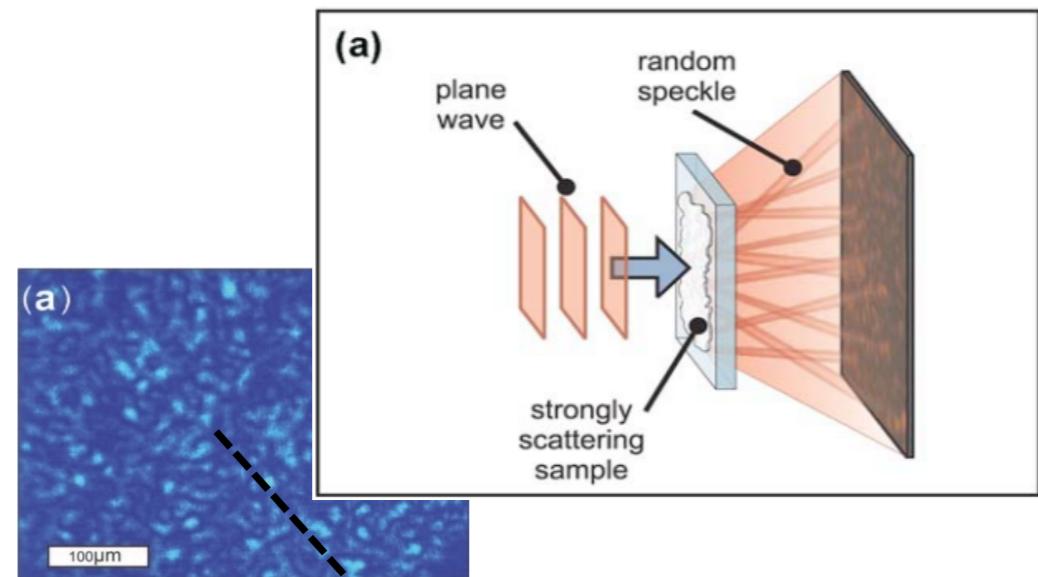
Optimization for focusing through complex media

IM Vellekoop and AP Mosk, Optics Letters, 32(16) 2007



Optimization for focusing through complex media

IM Vellekoop and AP Mosk, Optics Letters, 32(16) 2007

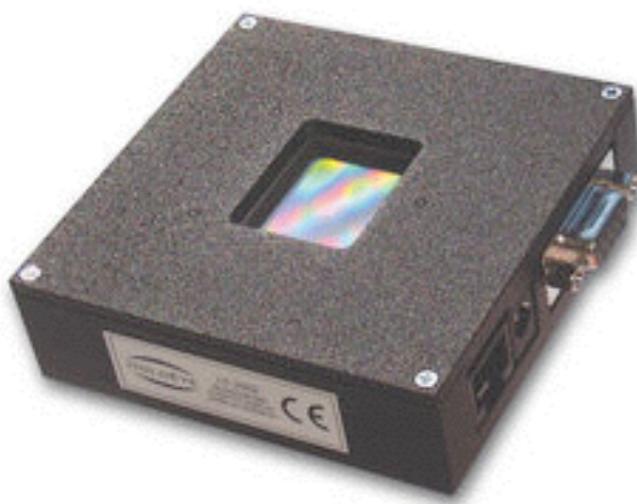


© 2007 Optical Society of America

It is possible to shape these modes in phase to obtain a constructive interference on a single speckle grain (Equivalent to phase-conjugation)

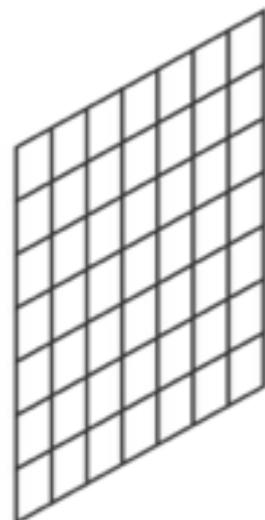
A more general approach : the transmission matrix

A more general approach : the transmission matrix



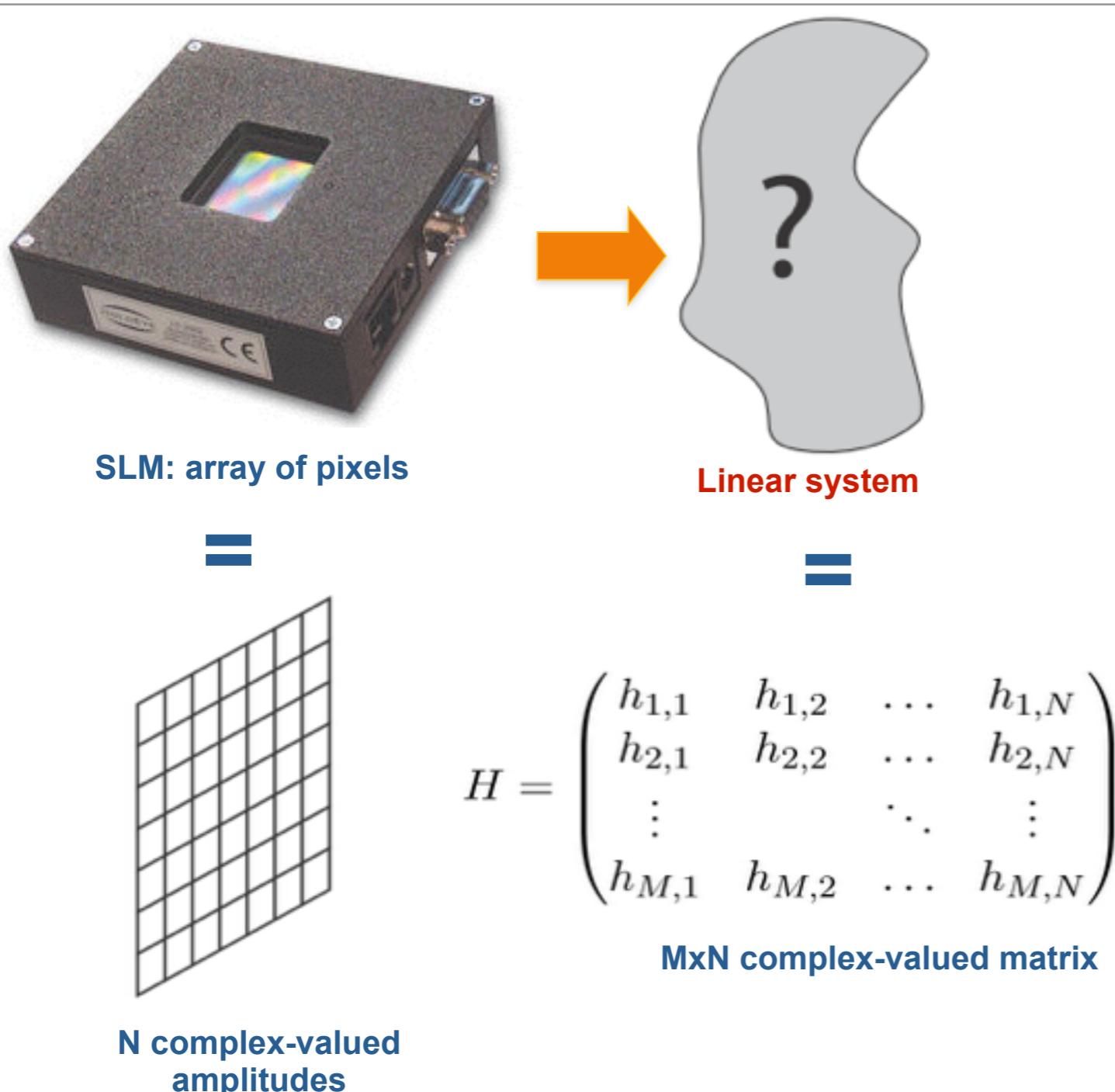
SLM: array of pixels

=

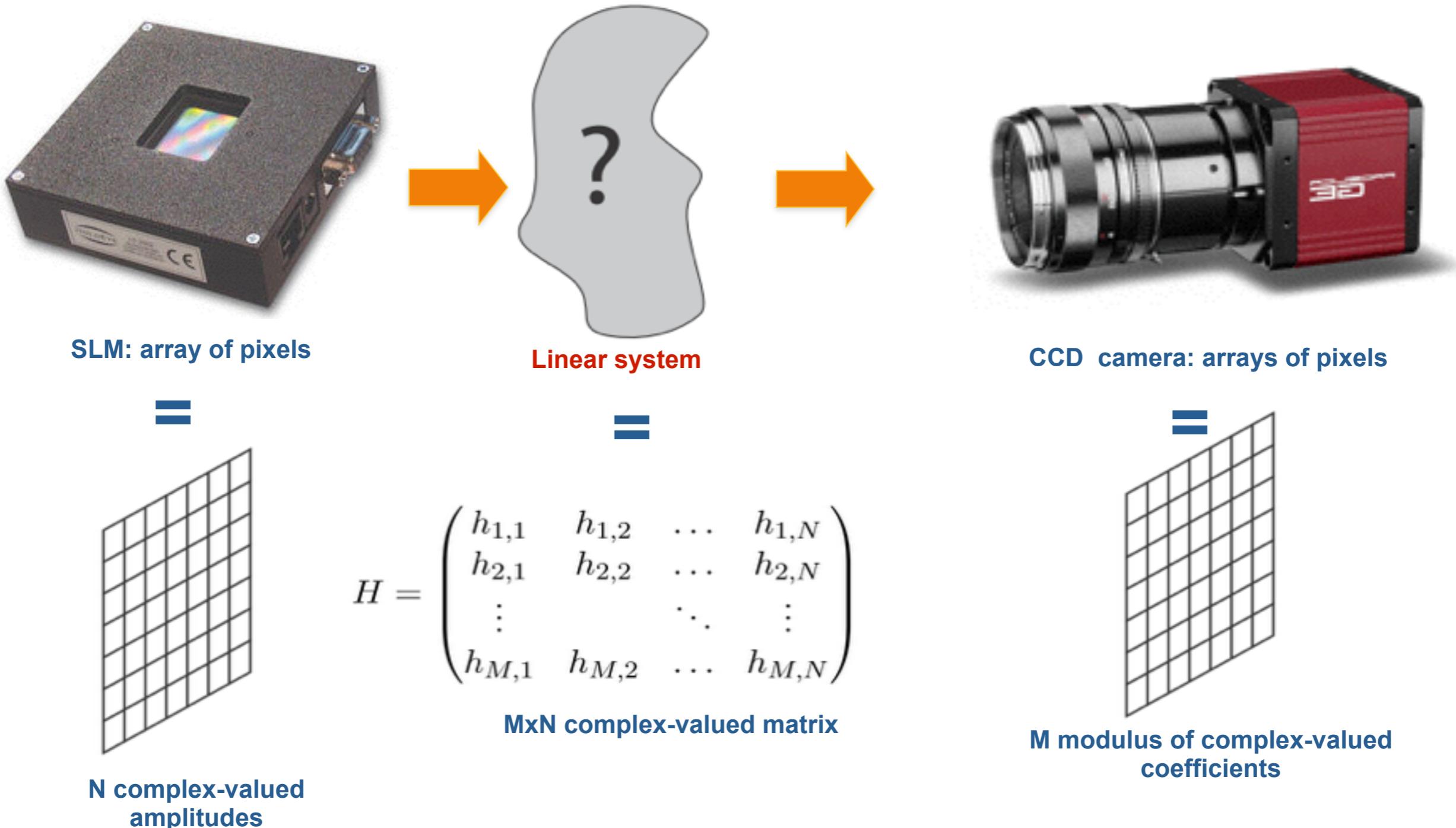


N complex-valued
amplitudes

A more general approach : the transmission matrix



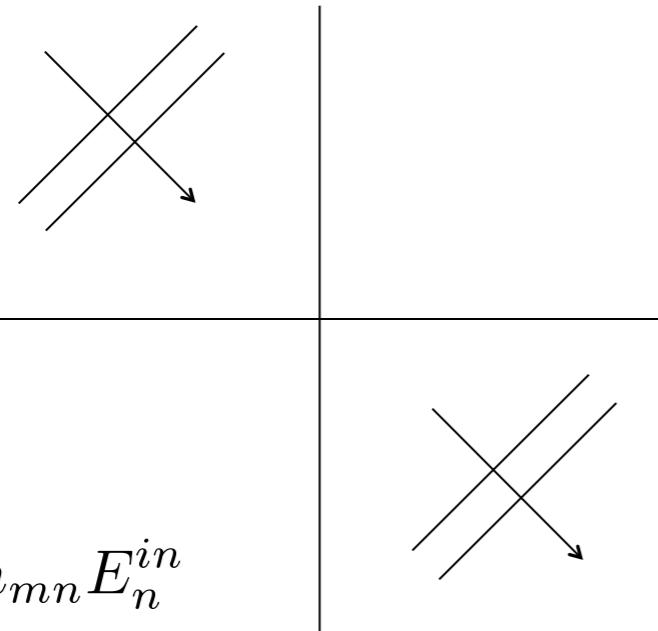
A more general approach : the transmission matrix



$$|E_m^{out}| = \left| \sum_n h_{mn} E_n^{in} \right|$$

A more general approach : the transmission matrix

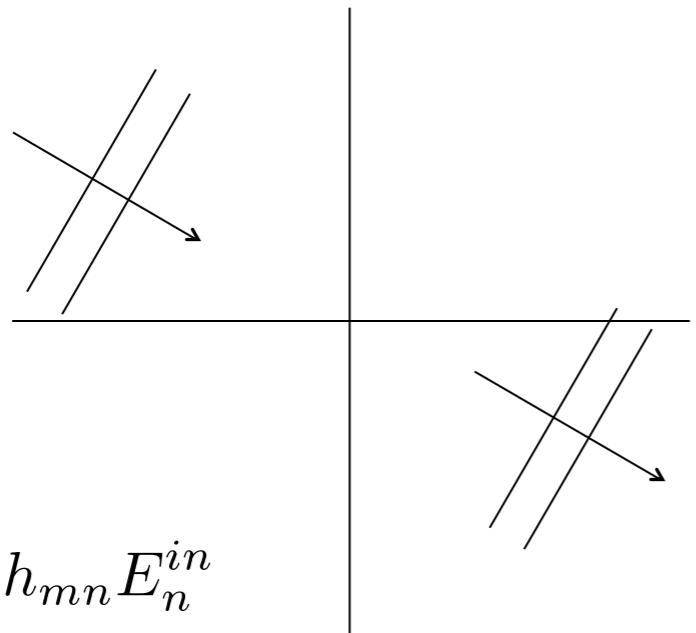
**free
field**



$$E_m^{out} = \sum_n^{1..N} h_{mn} E_n^{in}$$

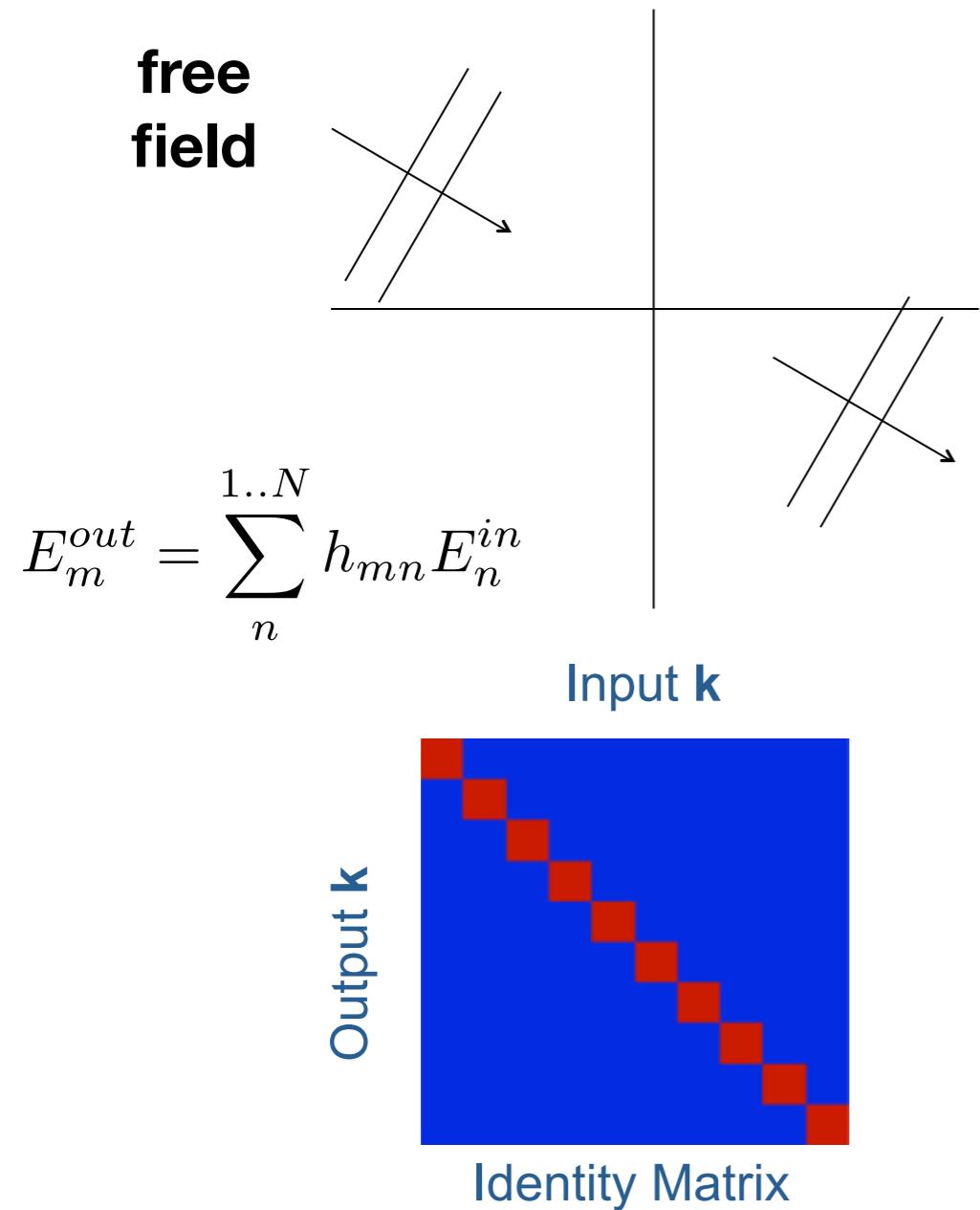
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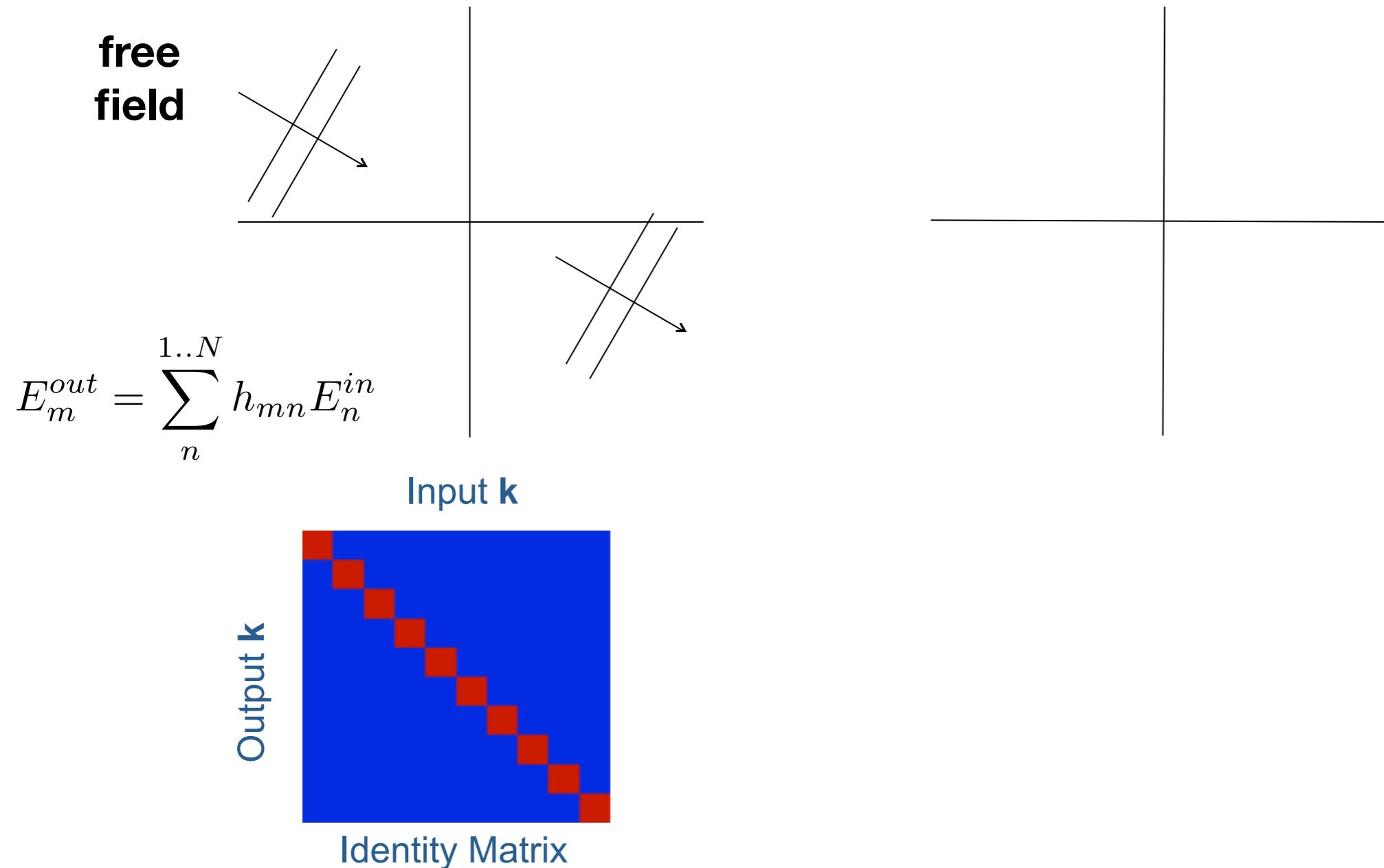


$$E_m^{out} = \sum_n^{1..N} h_{mn} E_n^{in}$$

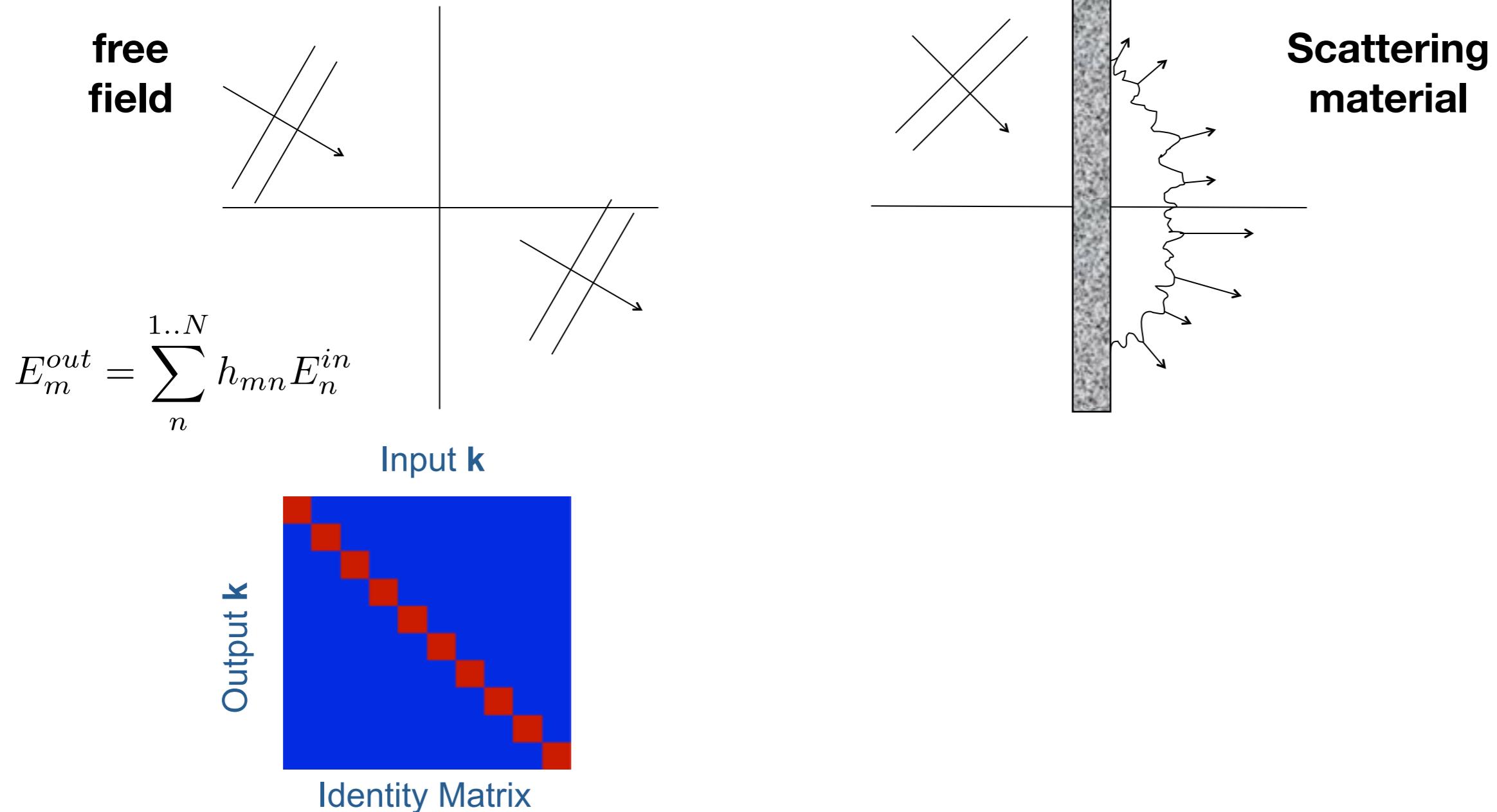
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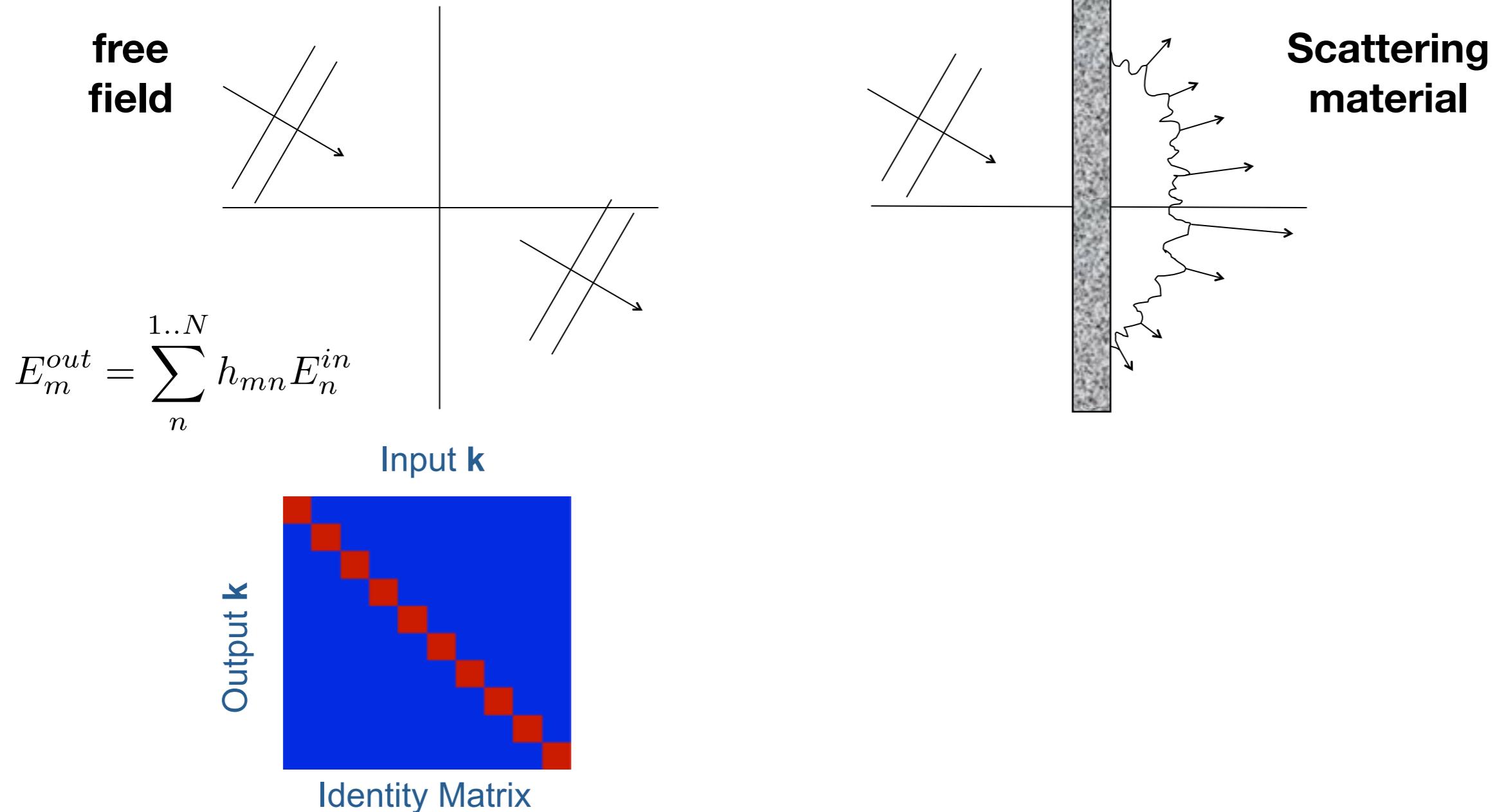
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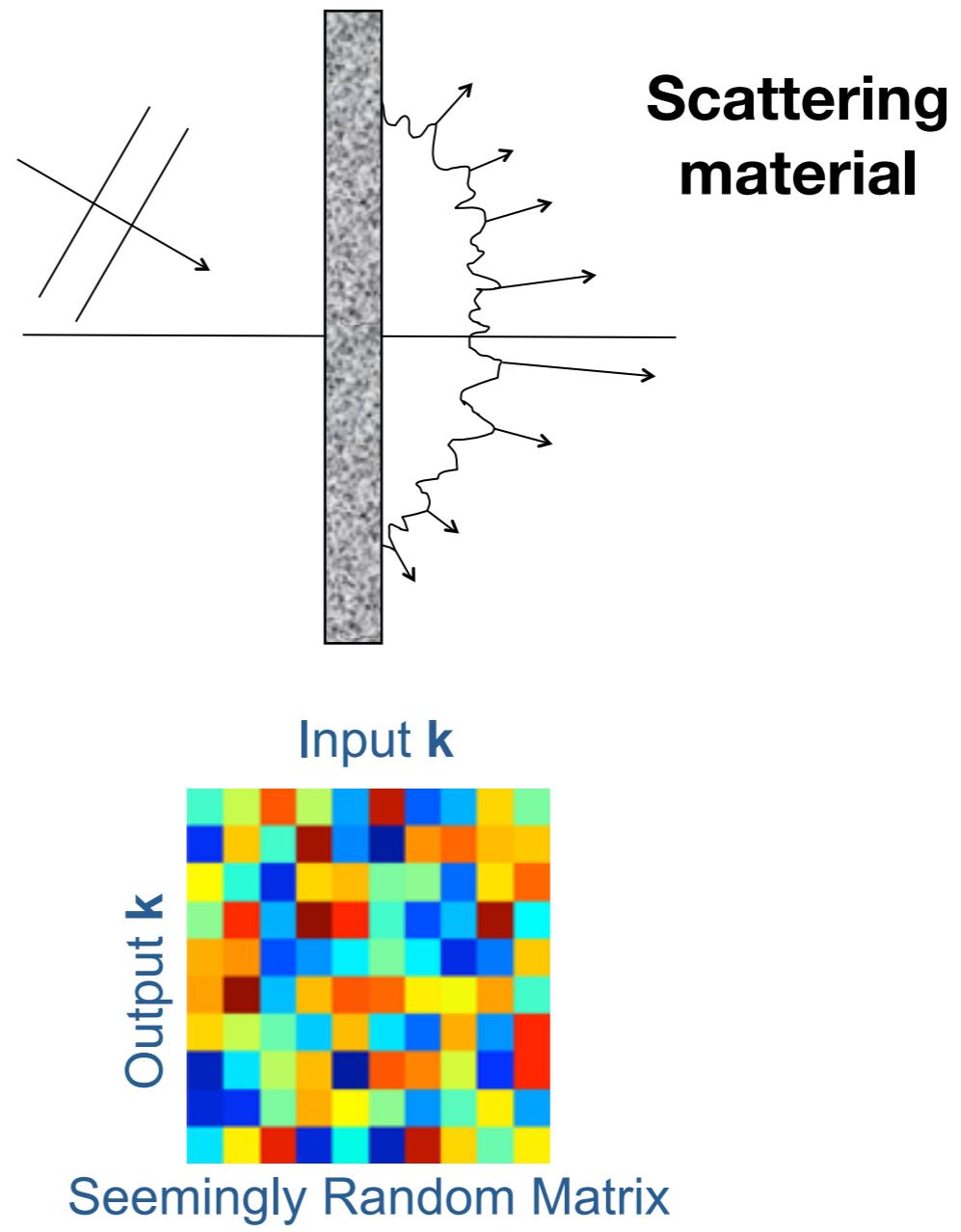
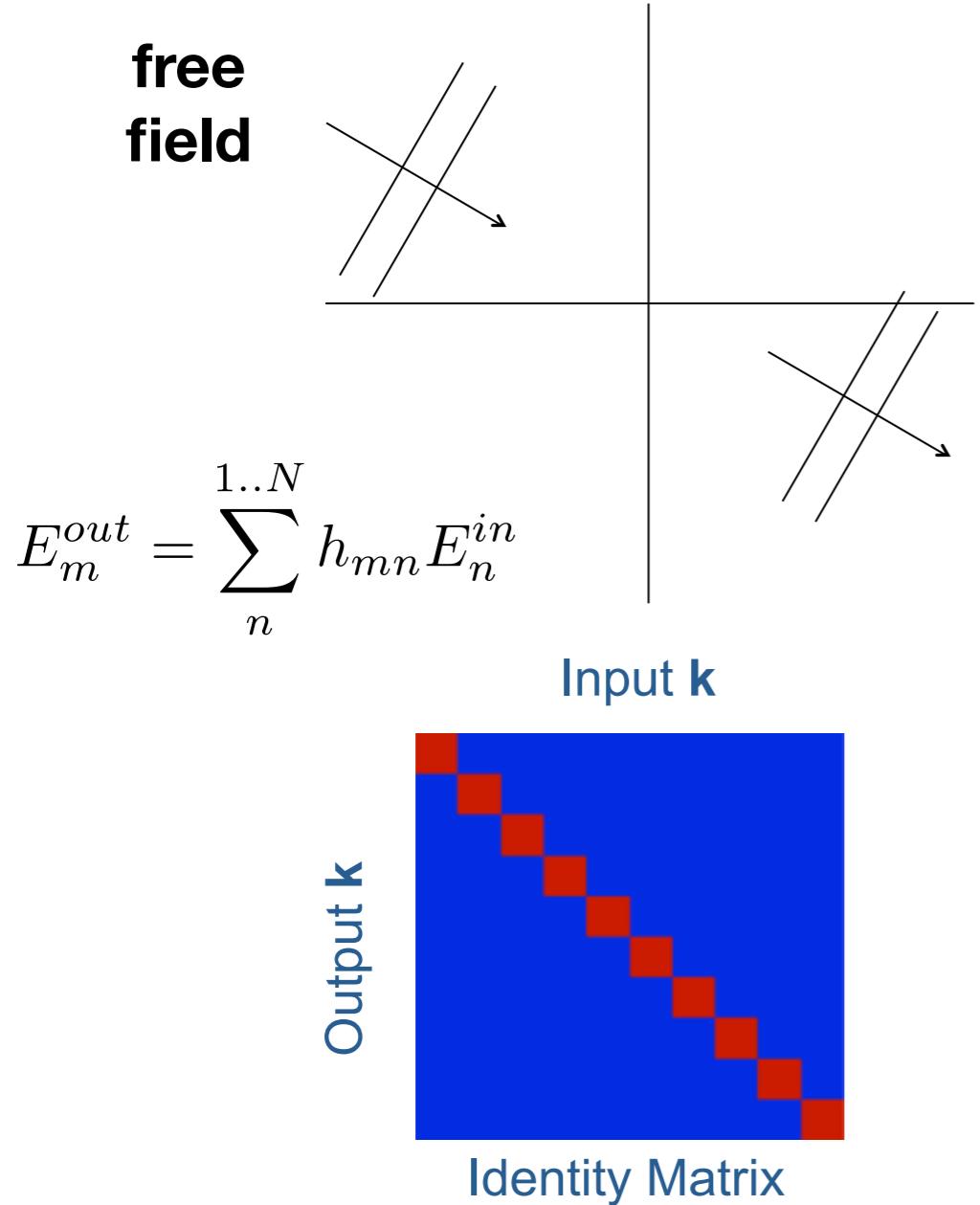
A more general approach : the transmission matrix



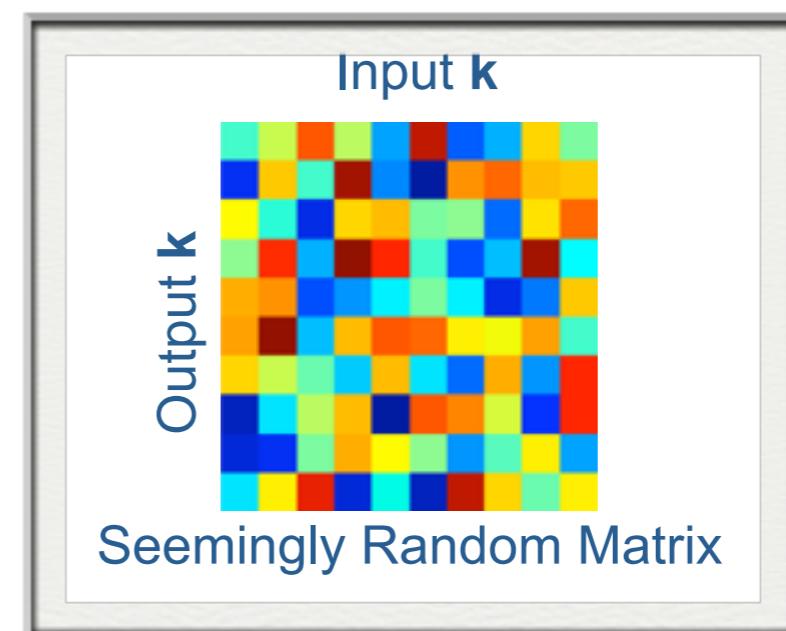
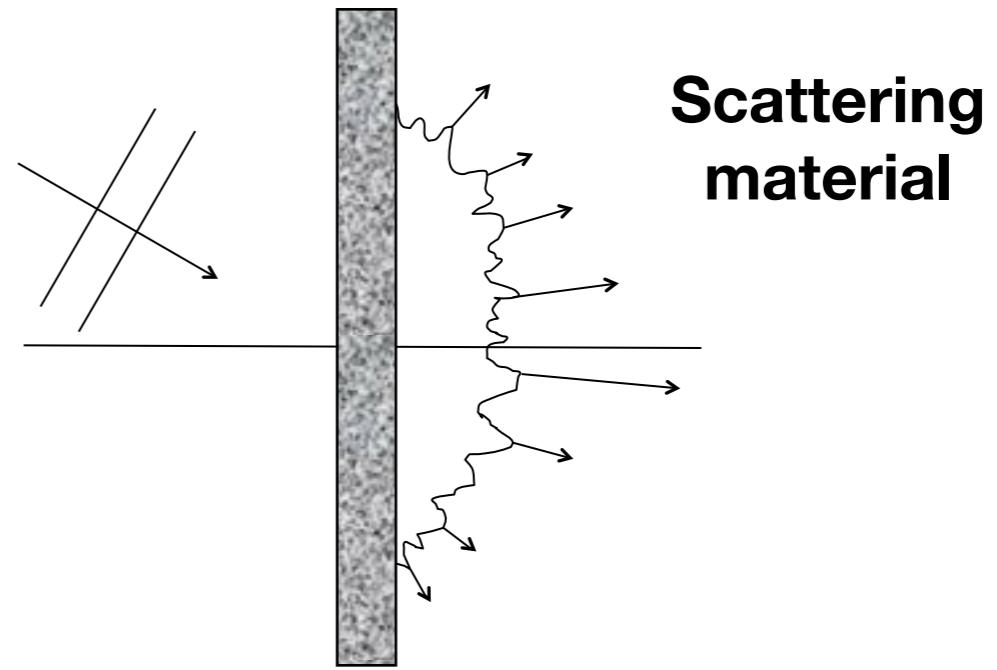
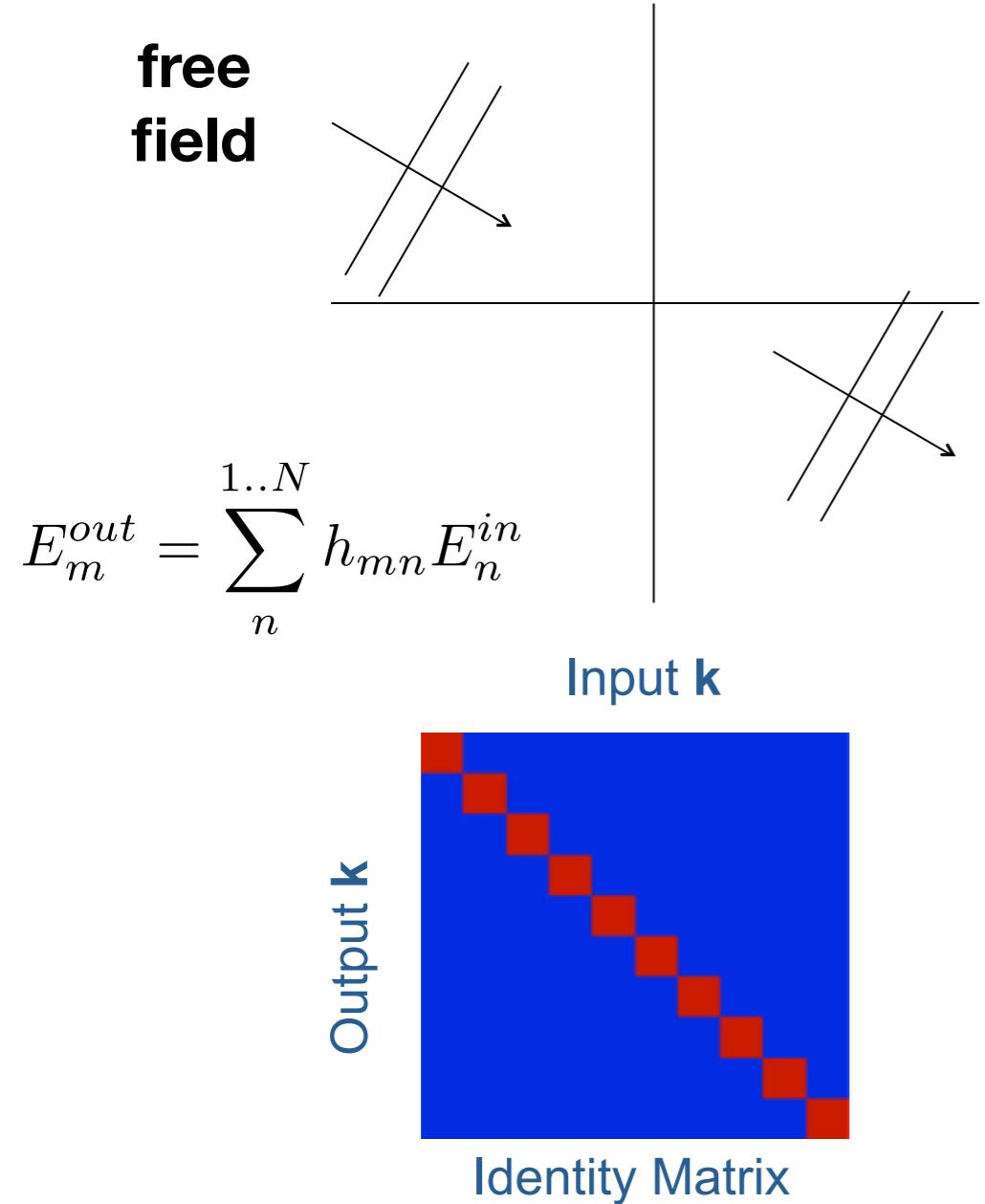
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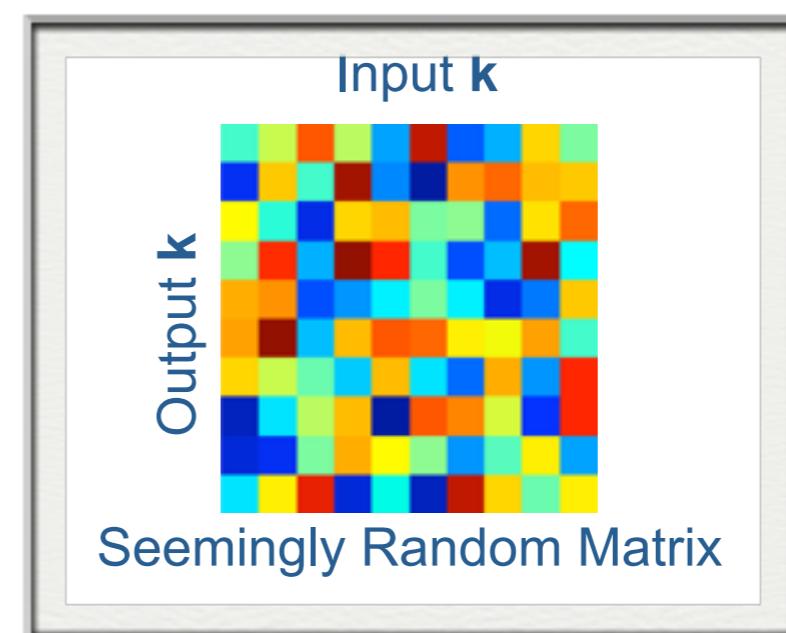
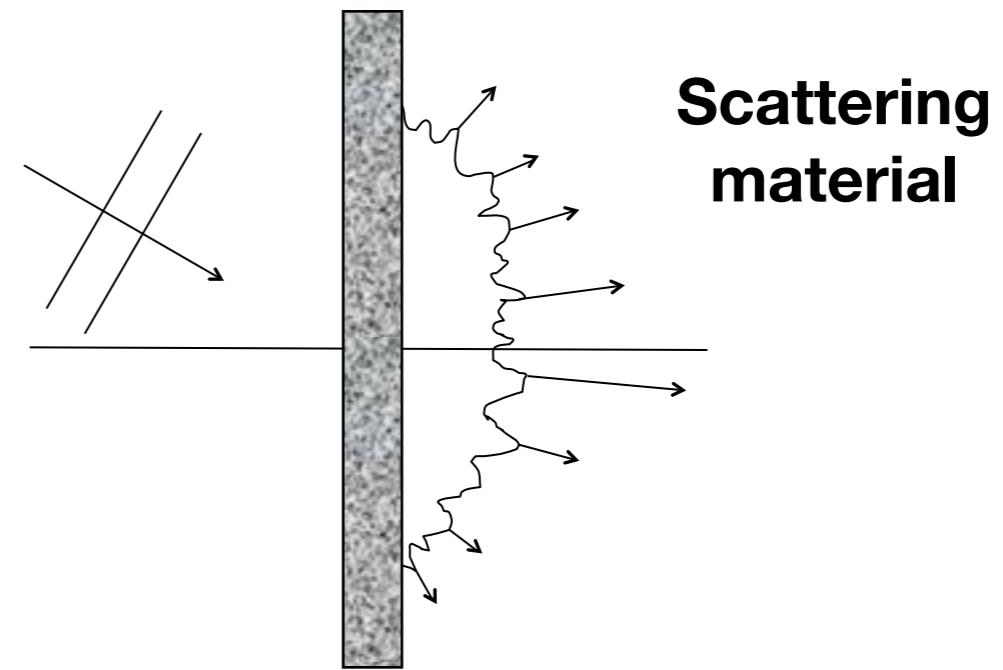
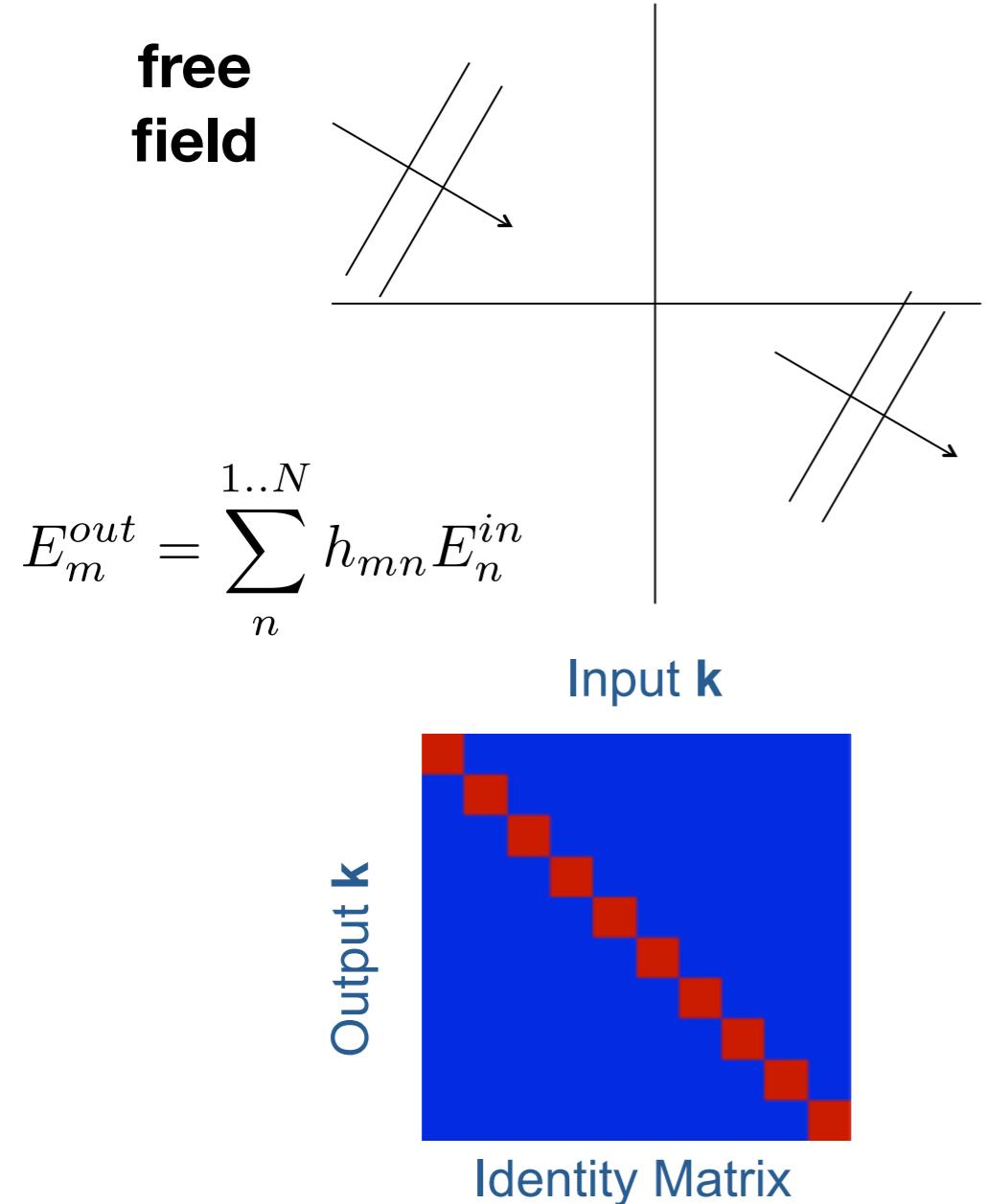
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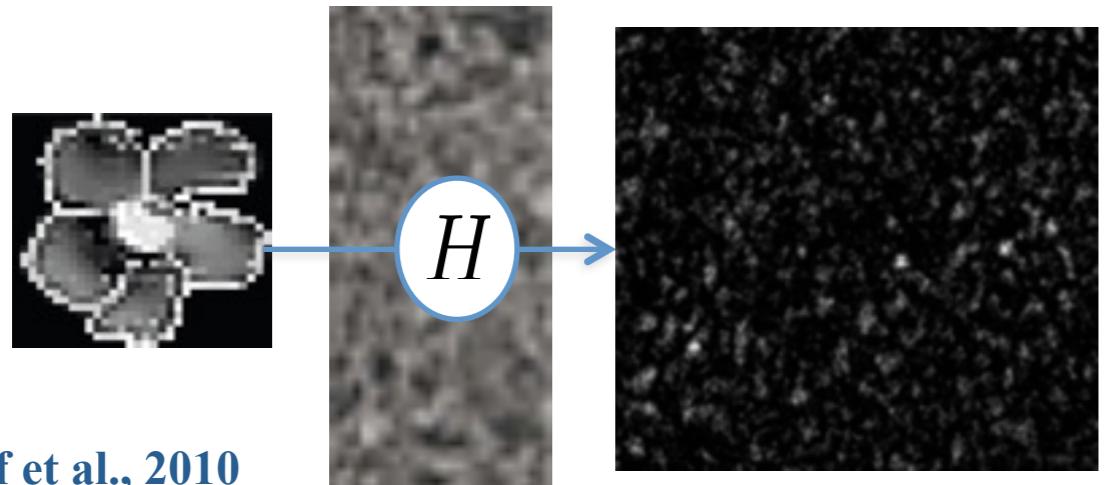


A more general approach : the transmission matrix



Gaussian iid measurements :
“optimal” for CS !

Exploiting H for imaging



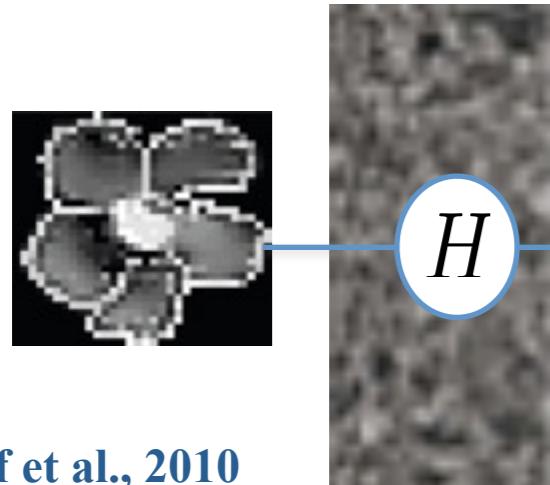
Linear Reconstruction

Tikhonov

$$(H^\dagger H + \sigma I)^{-1} H^\dagger$$

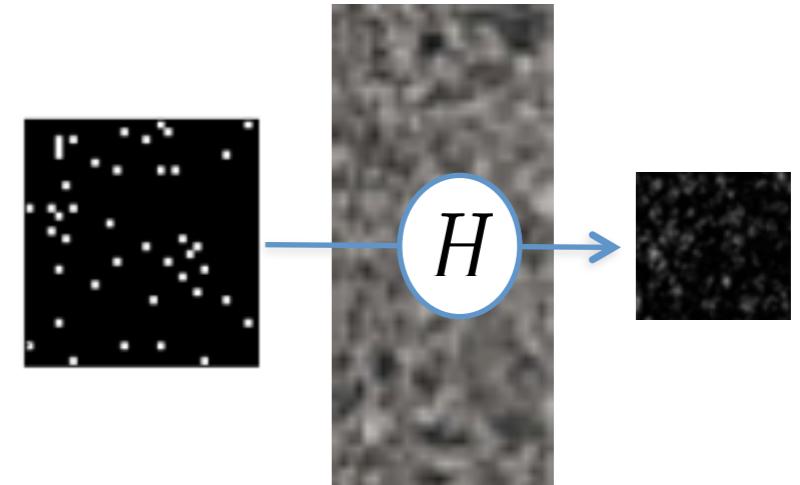


Exploiting H for imaging



Popoff et al., 2010

**Sparse
image**



Linear Reconstruction

Tikhonov

$$(H^\dagger H + \sigma I)^{-1} H^\dagger$$

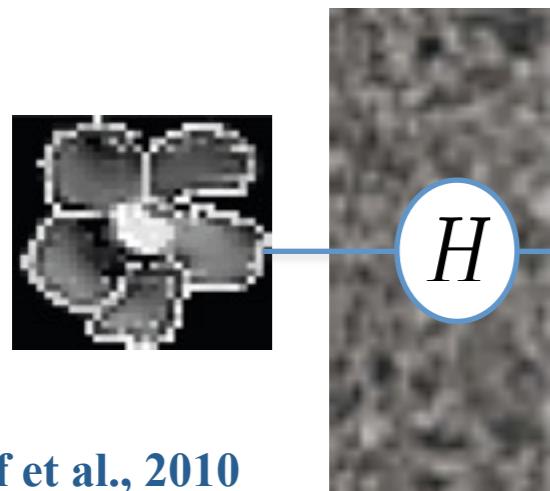


Non-linear Reconstruction

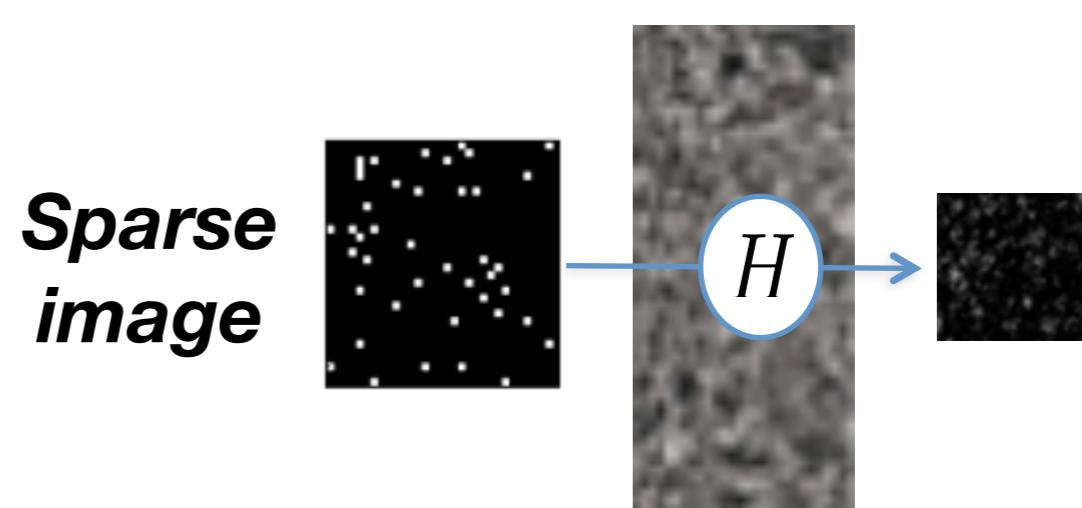
Sparse reconstruction (l1 or l0)

$$u = \arg \min_u \|x - H u\|_2^2 + \lambda \|u\|_1$$

Exploiting H for imaging



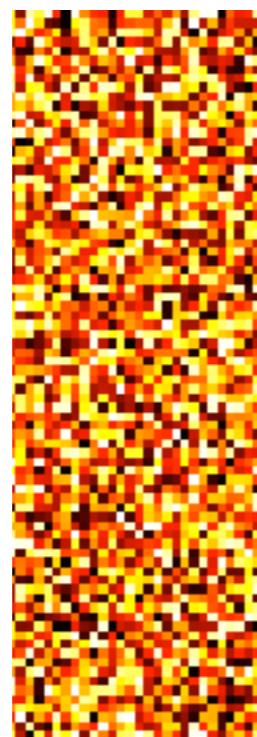
Popoff et al., 2010



Sparse image

Linear Reconstruction

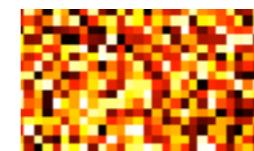
H



At least as many measurement pixels as input pixels

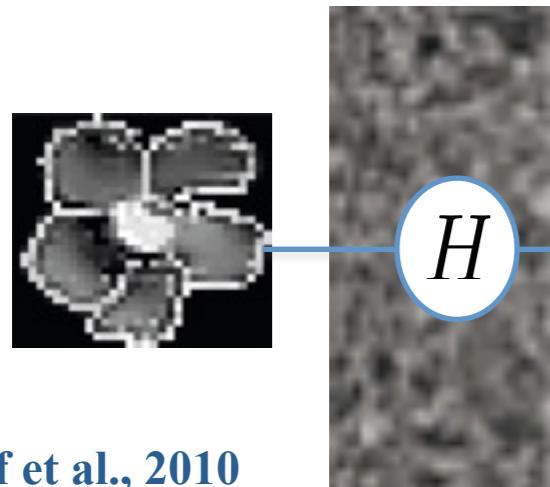
Non-linear Reconstruction

H



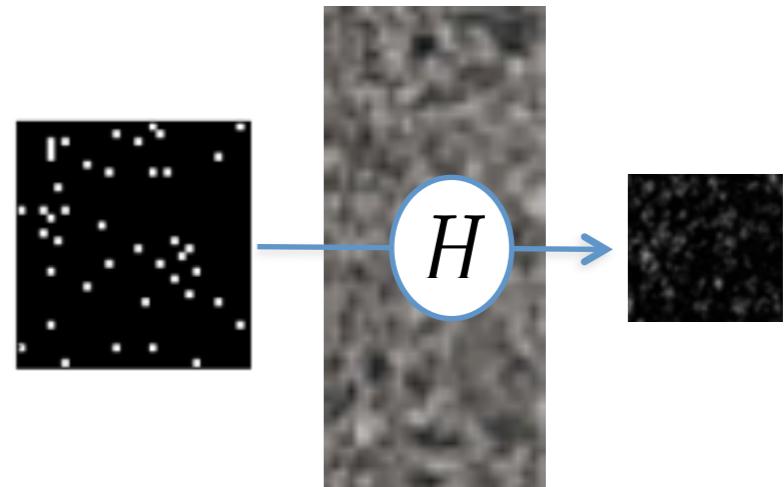
Number of measurement pixels driven by sparsity (\ll input pixels)

Exploiting H for imaging



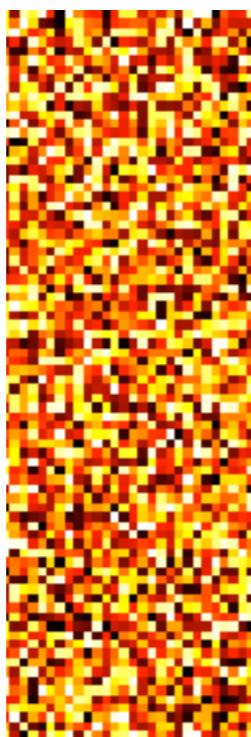
Popoff et al., 2010

Sparse image



Linear Reconstruction

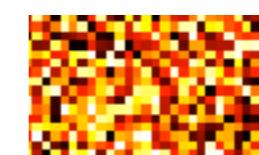
H



At least as many measurement pixels as input pixels

Non-linear Reconstruction

H

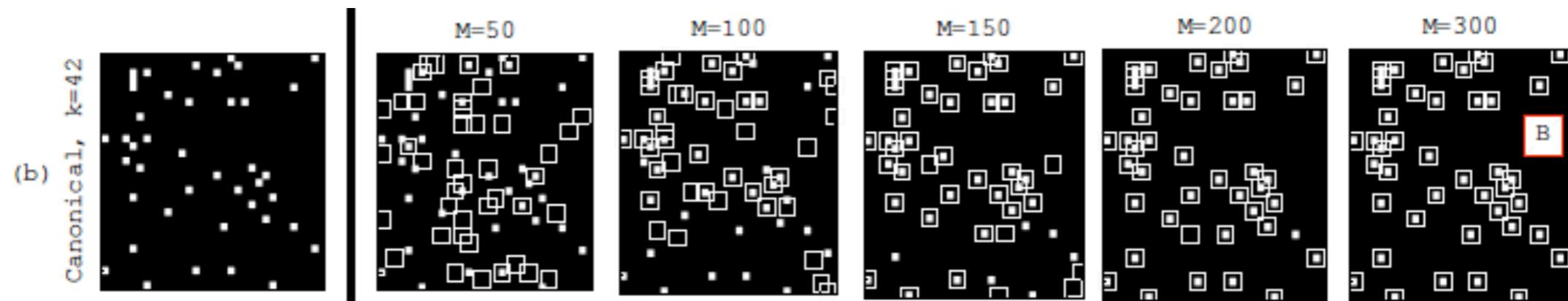


Number of measurement pixels driven by sparsity (\ll input pixels)

Compressive imaging with scattering media

original image
1024 pixels

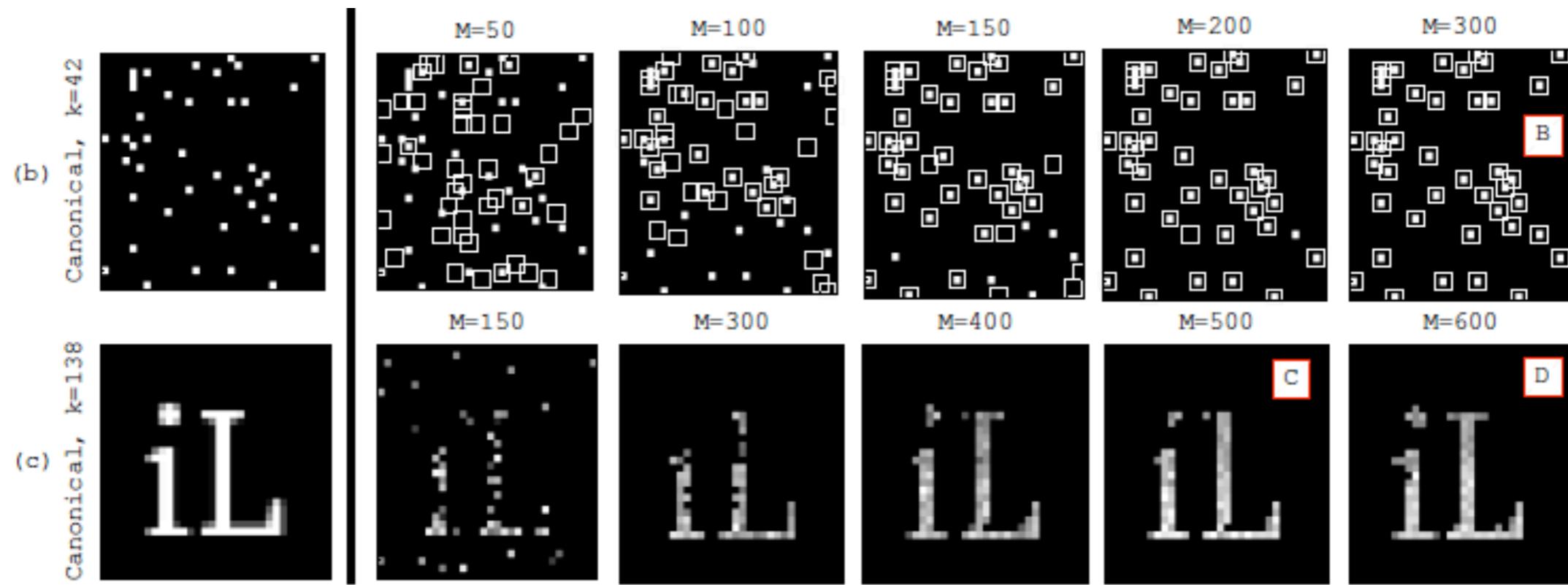
number of pixels M used for reconstruction



Compressive imaging with scattering media

original image
1024 pixels

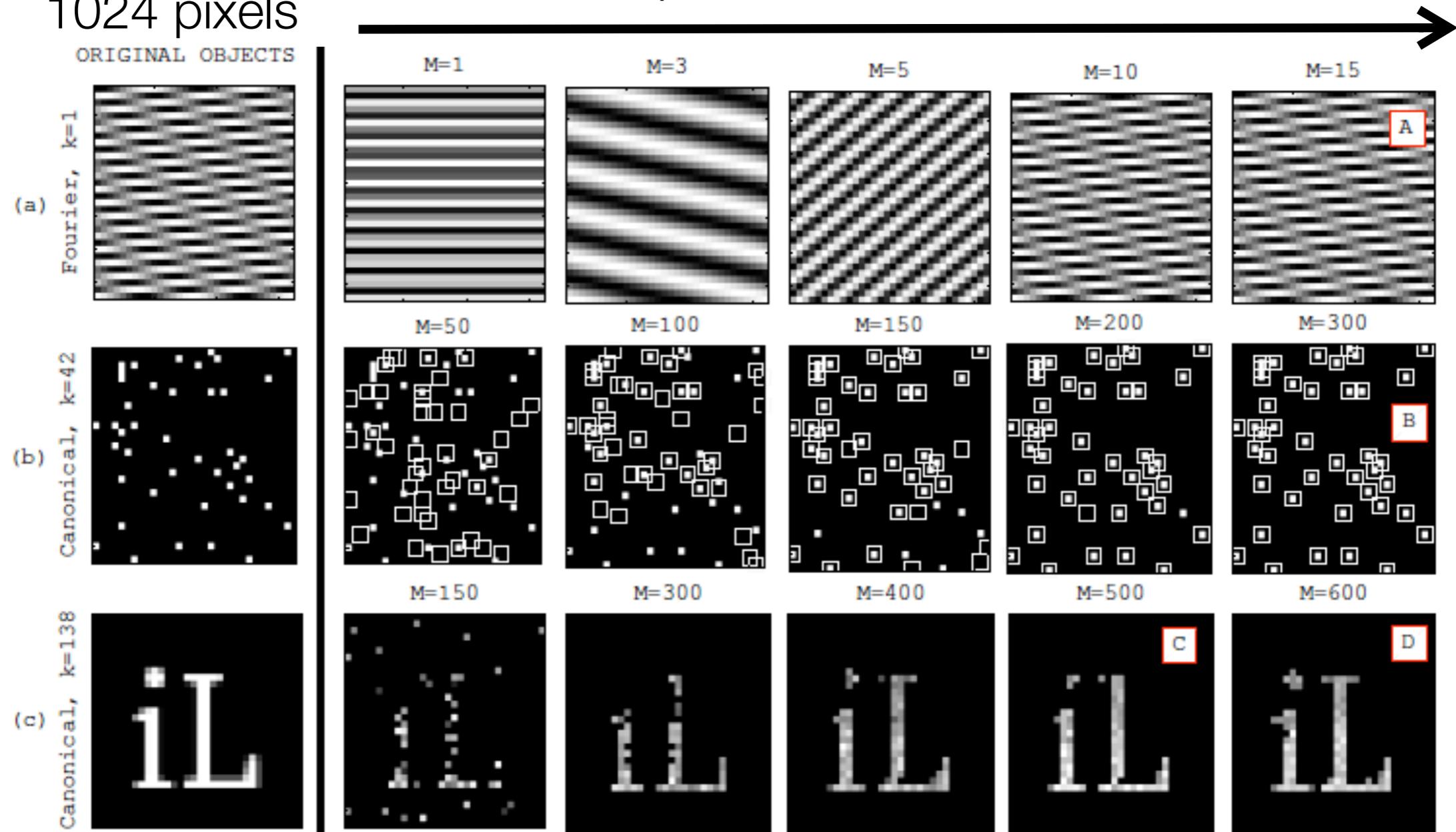
number of pixels M used for reconstruction



Compressive imaging with scattering media

original image
1024 pixels

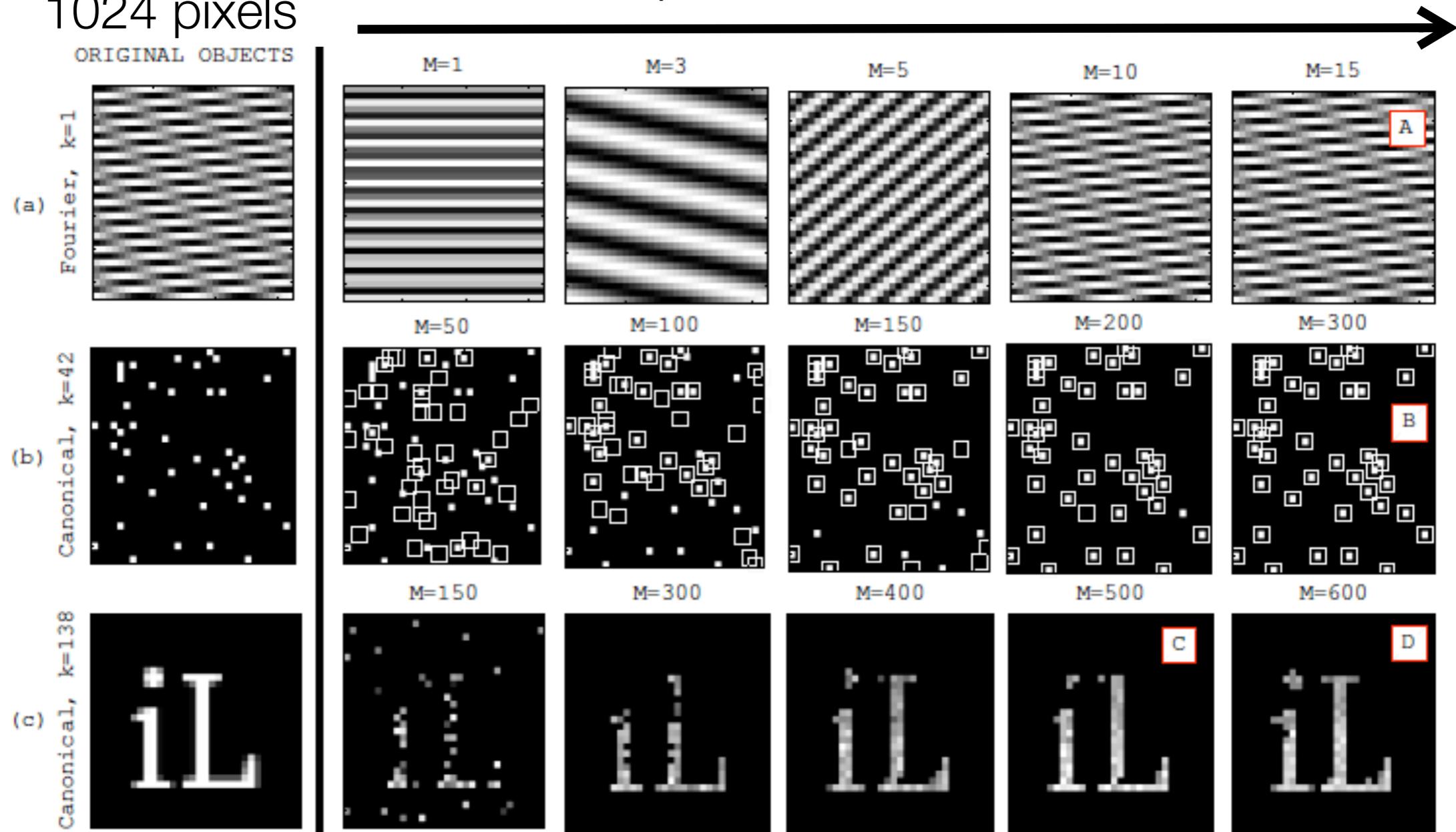
number of pixels M used for reconstruction



Compressive imaging with scattering media

original image
1024 pixels

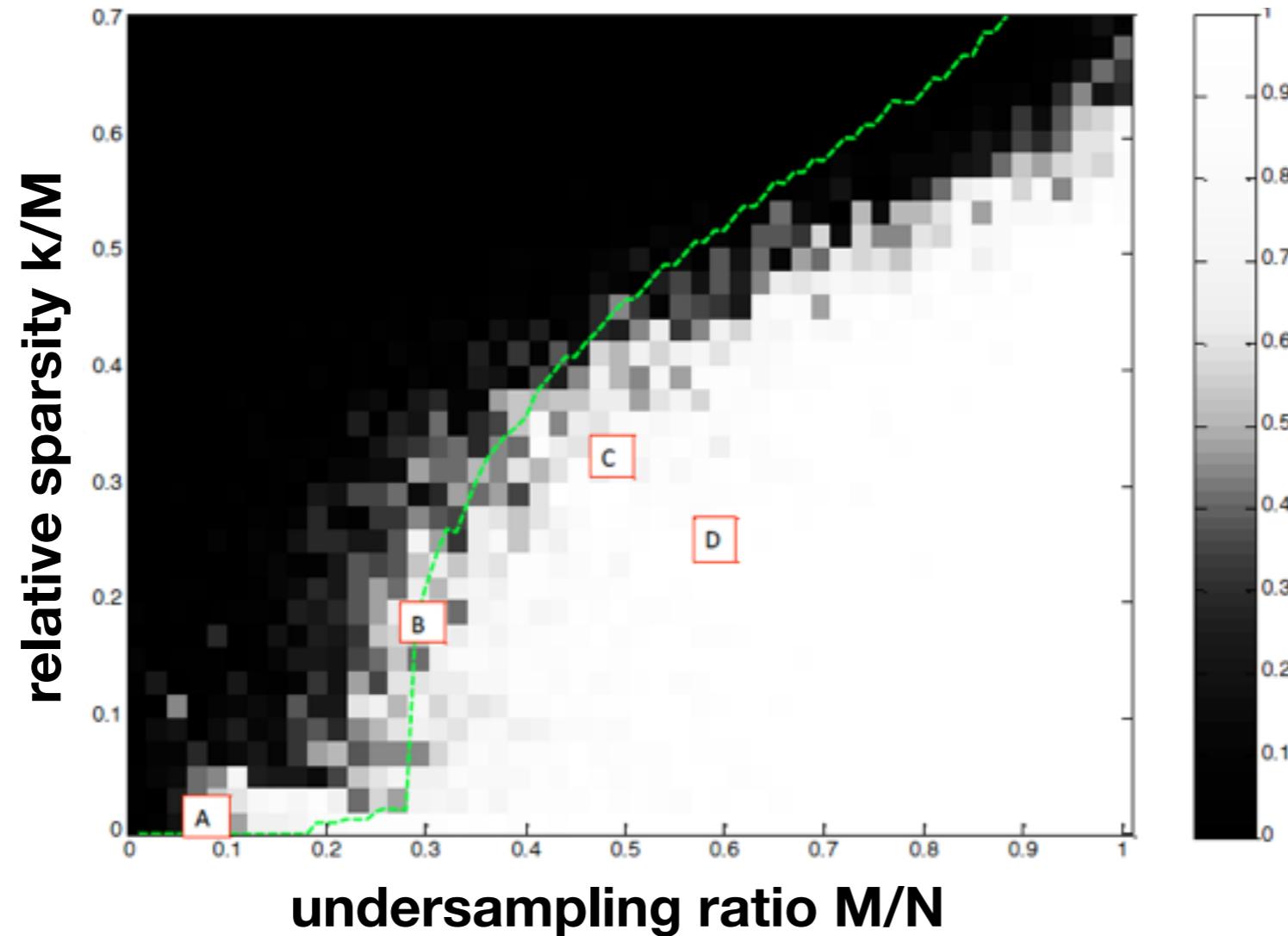
number of pixels M used for reconstruction



Each pixel provides information about the whole image

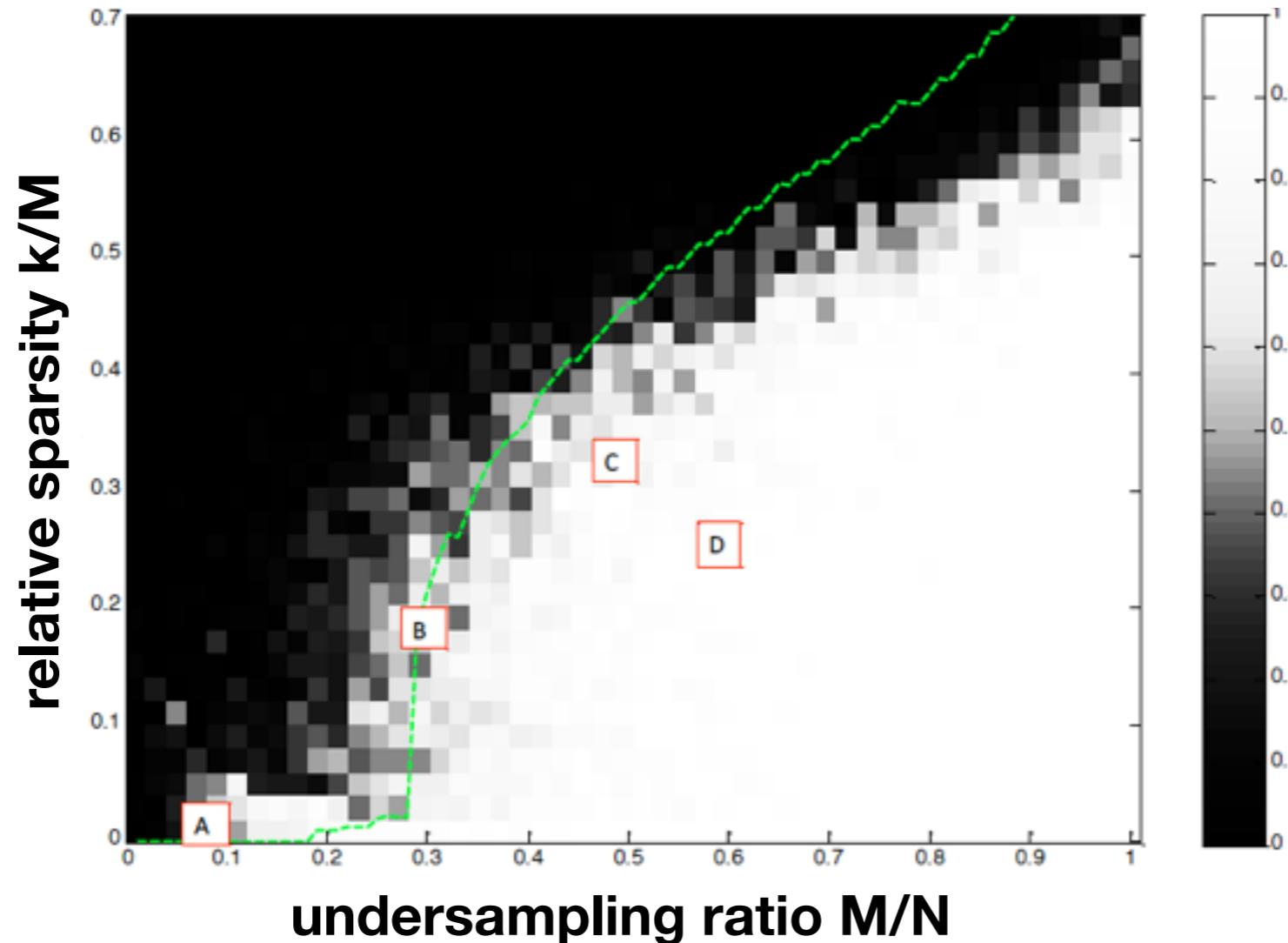
Compressive imaging with scattering media

Probability of success for recovery (MMV with 3 observations)



Compressive imaging with scattering media

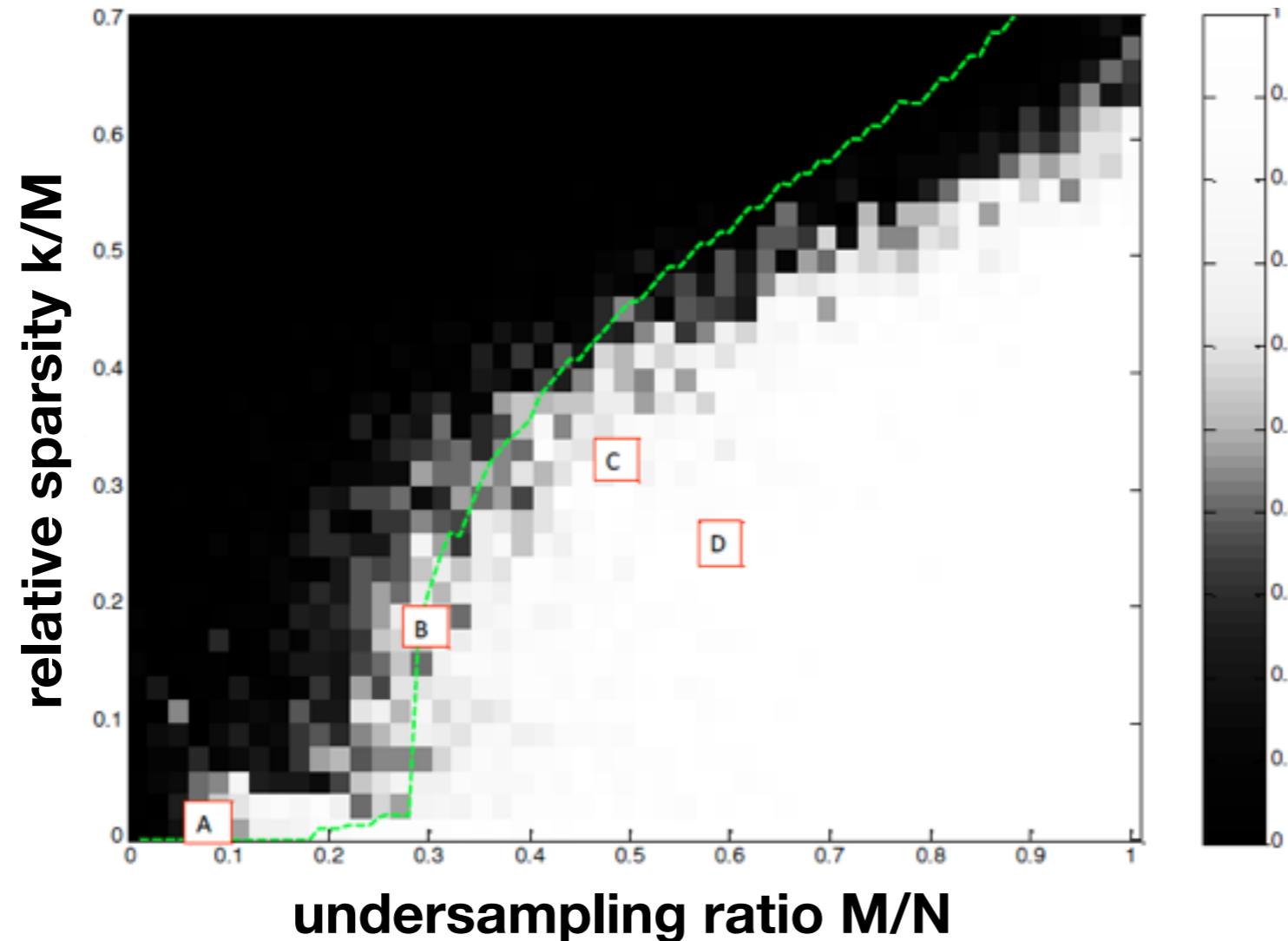
Probability of success for recovery (MMV with 3 observations)



about 10^5 experiments needed !
(measurements are fast ! medium is stable only for ~ 30 min)

Compressive imaging with scattering media

Probability of success for recovery (MMV with 3 observations)

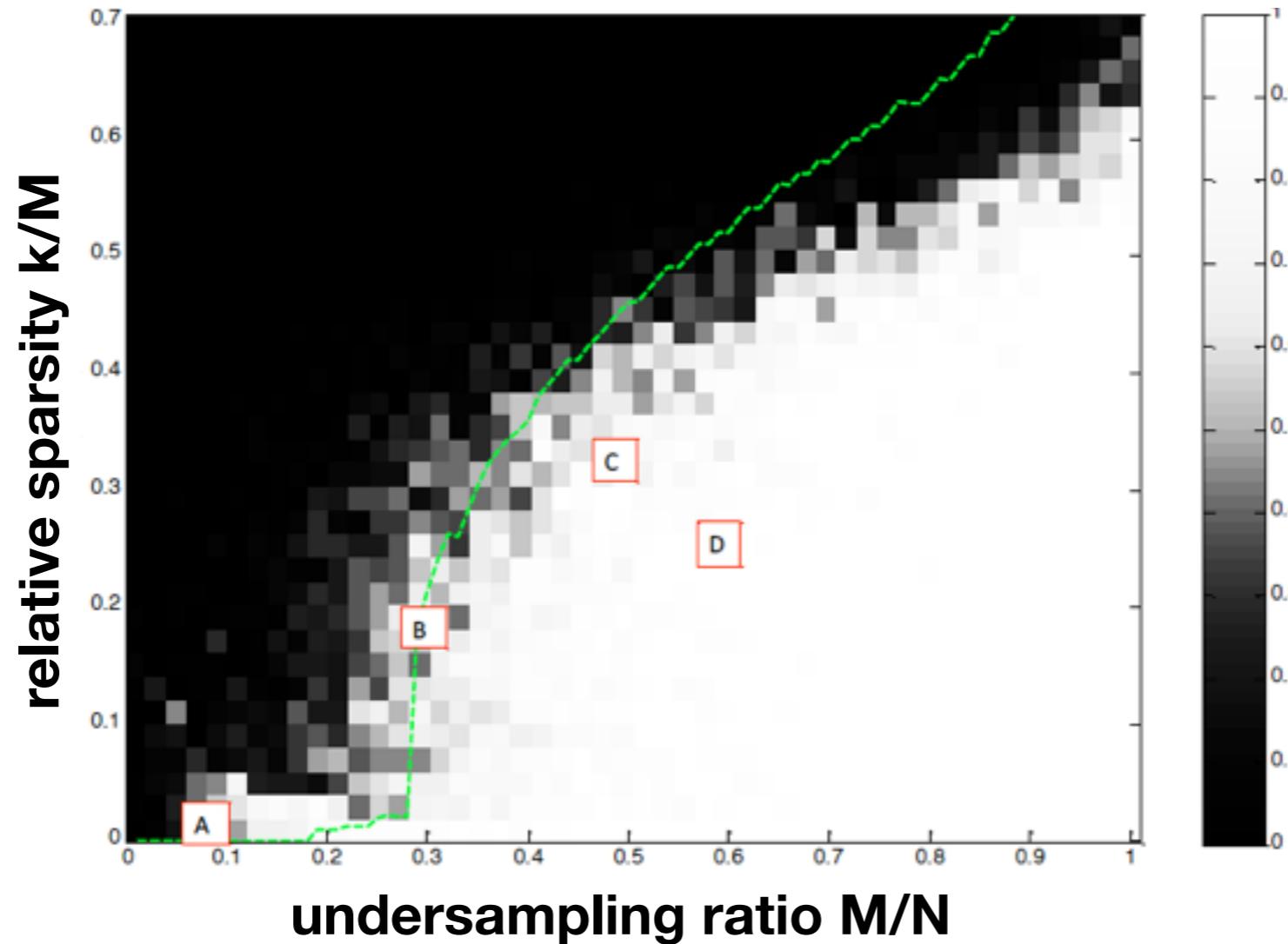


about 10^5 experiments needed !
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Why is it different from the standard «a la Donoho-Tanner» phase transition ?

Compressive imaging with scattering media

Probability of success for recovery (MMV with 3 observations)



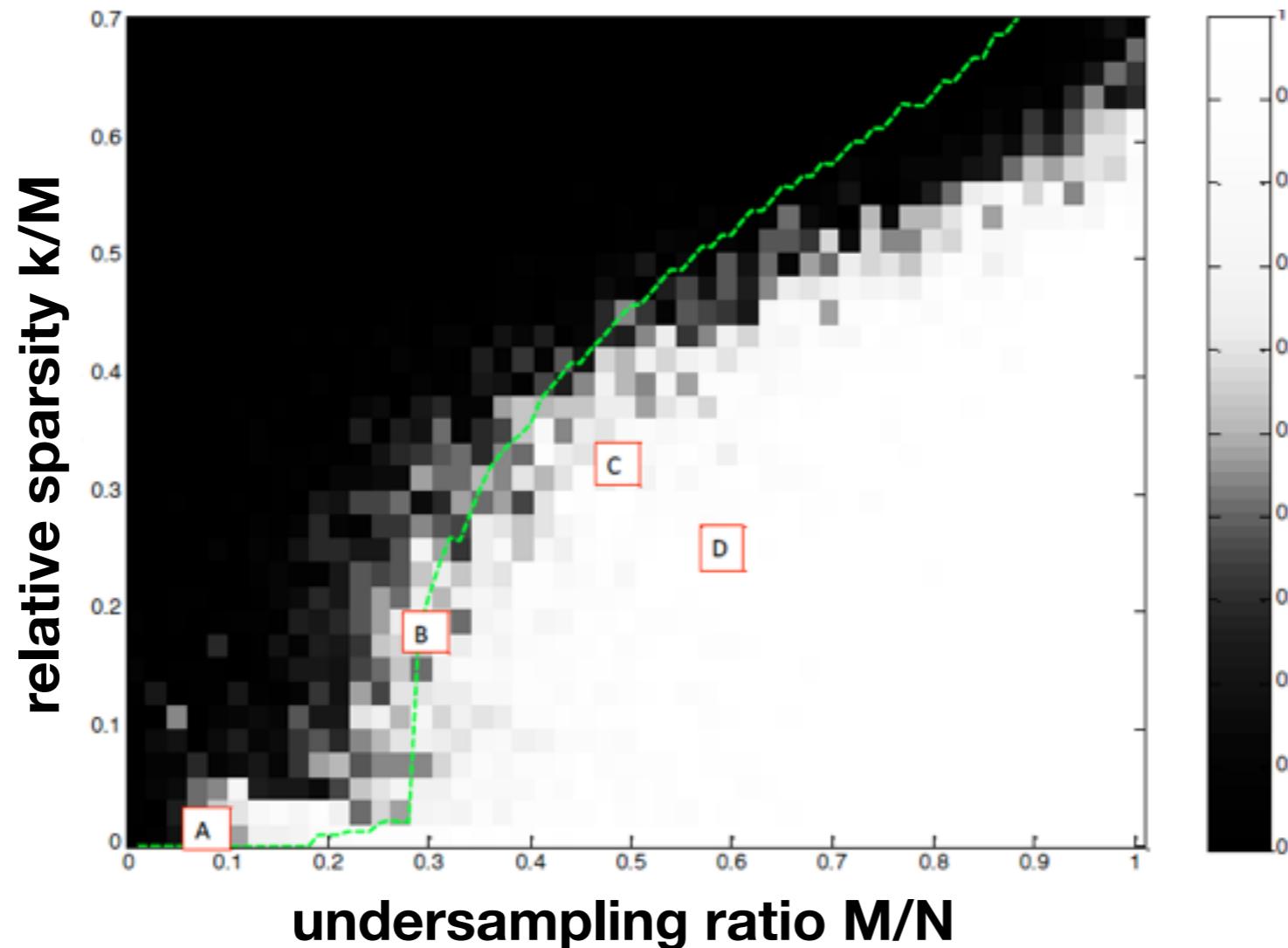
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Compressive imaging with scattering media

Probability of success for recovery (MMV with 3 observations)



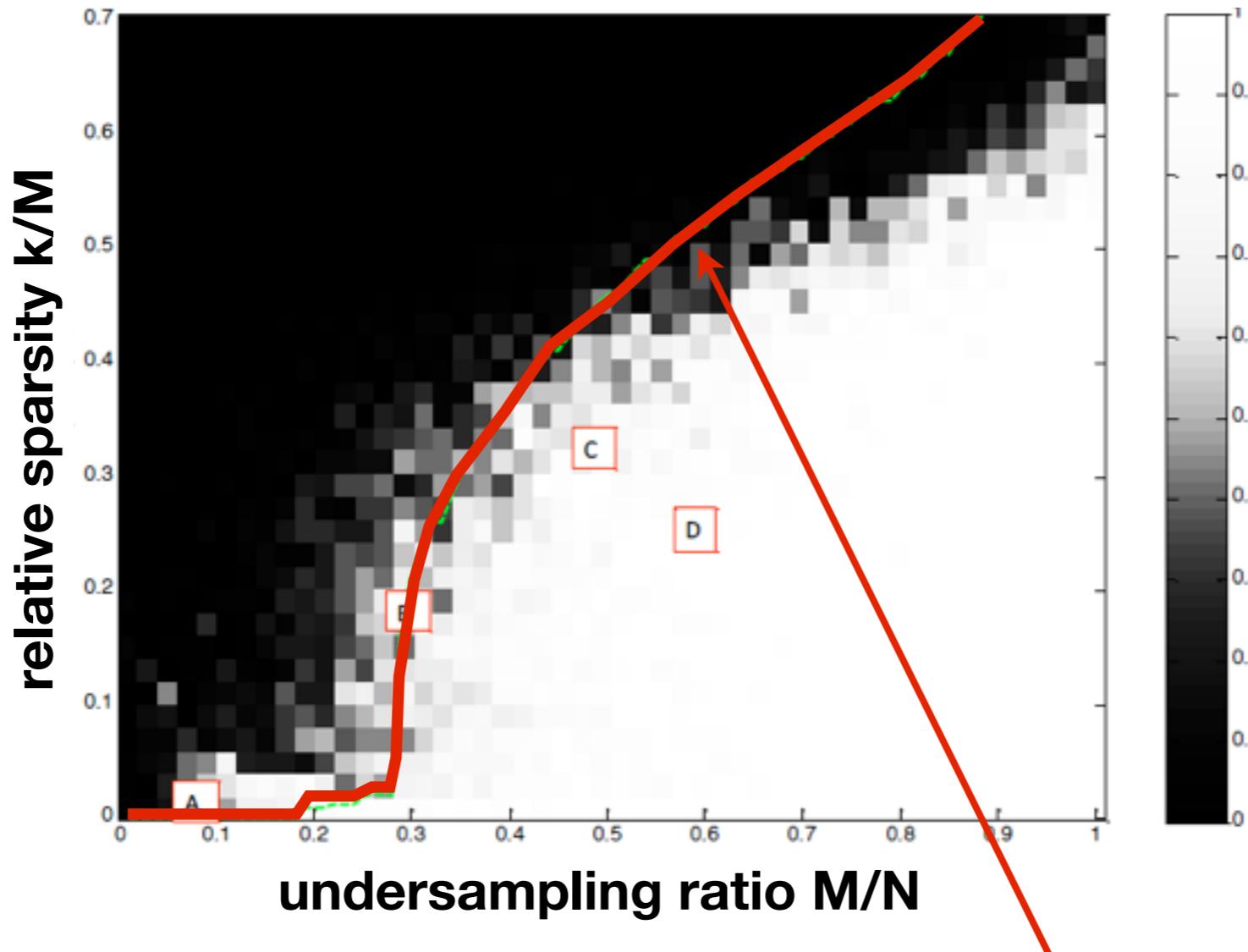
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Why is it different from the standard «a la Donoho-Tanner» phase transition ?

- algorithm used is SOMP (group- ℓ_0), not a ℓ_1 minimizer
- noise in the measurement and in the calibration (estimation of H)

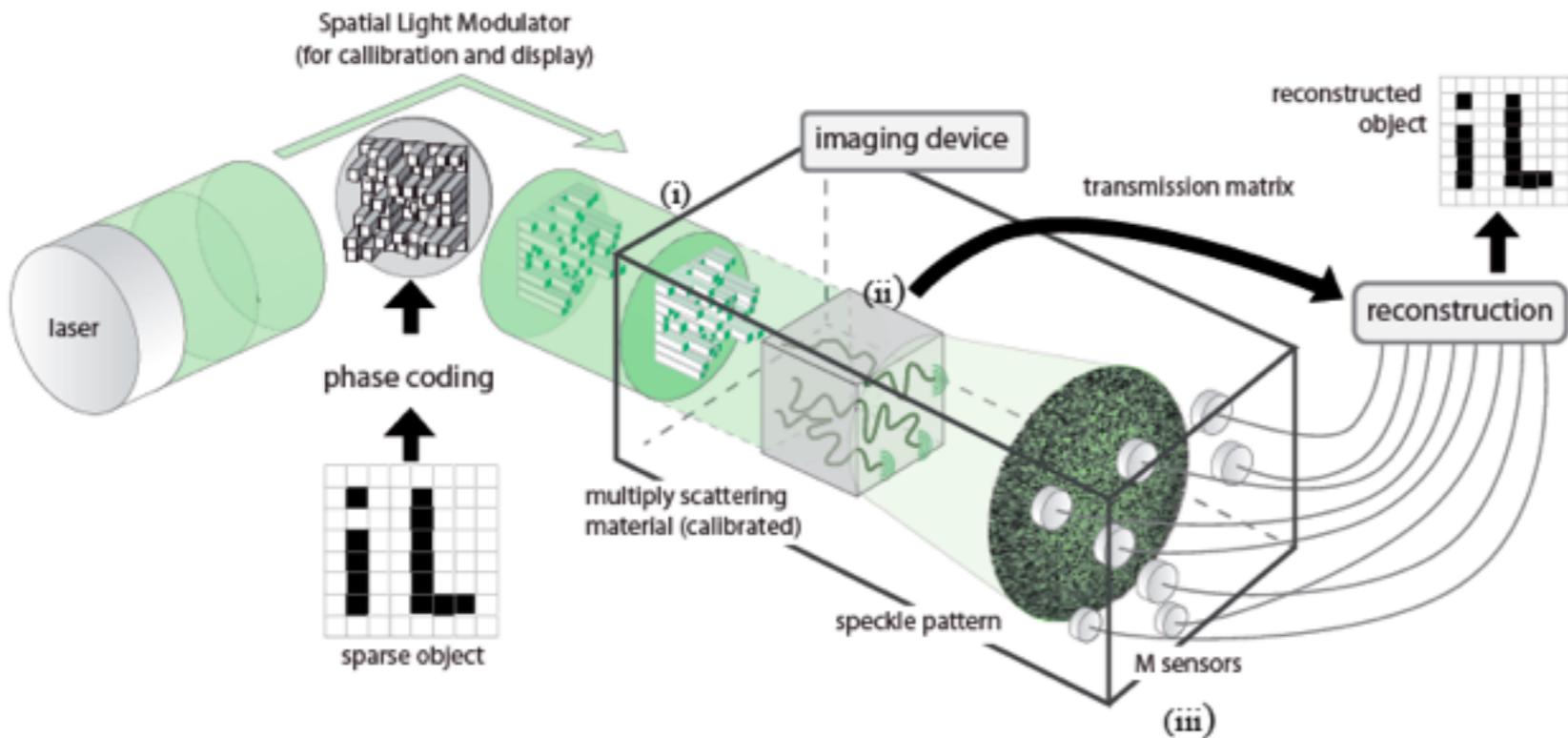
Compressive imaging with scattering media

Probability of success for recovery (MMV with 3 observations)

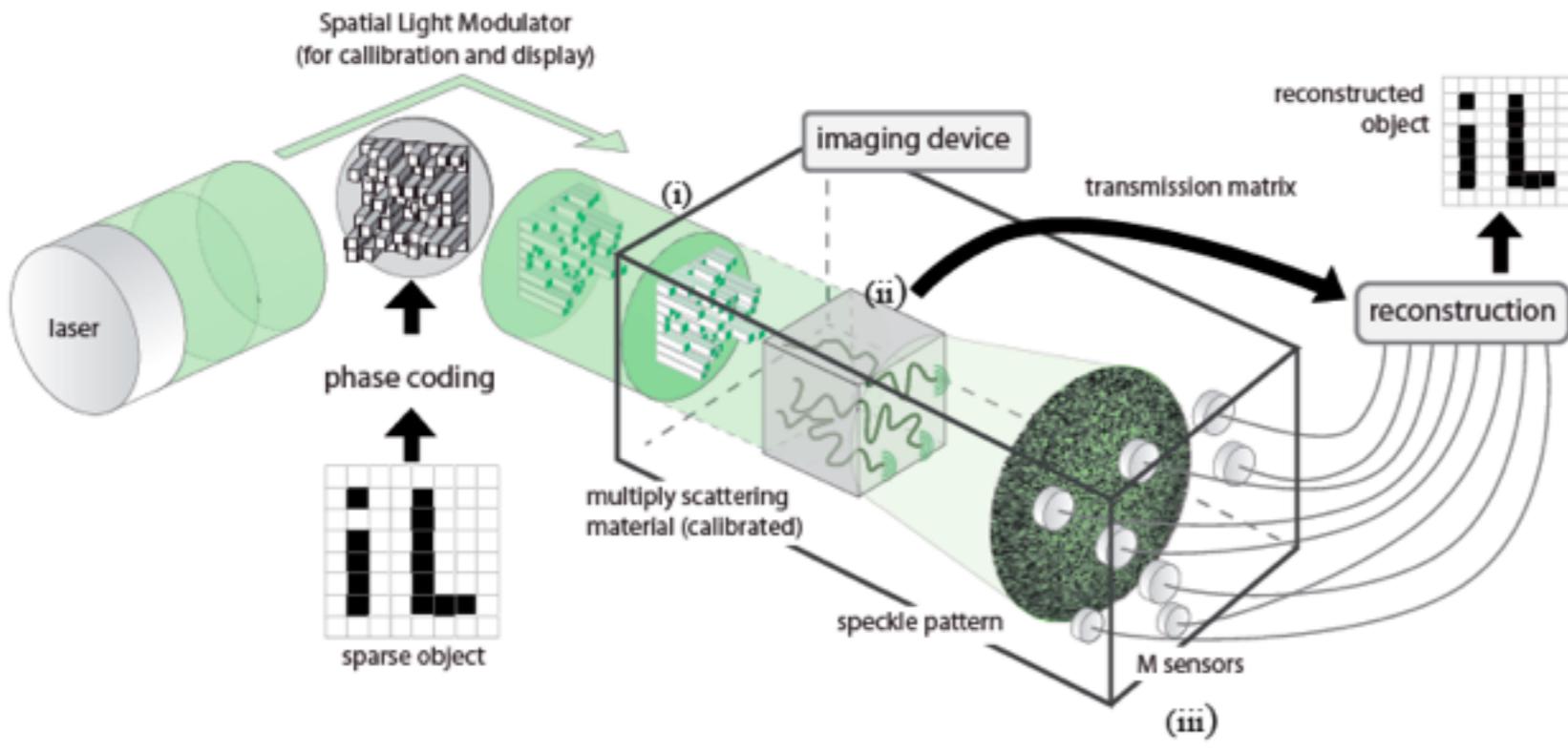


Transition curve obtained by numerical simulation with $y = Mx + e$
with M and e with gaussian iid entries
variance of $e = 3\%$ of variance of M
same procedure : error added at calibration and measurement

Compressive imaging with scattering media

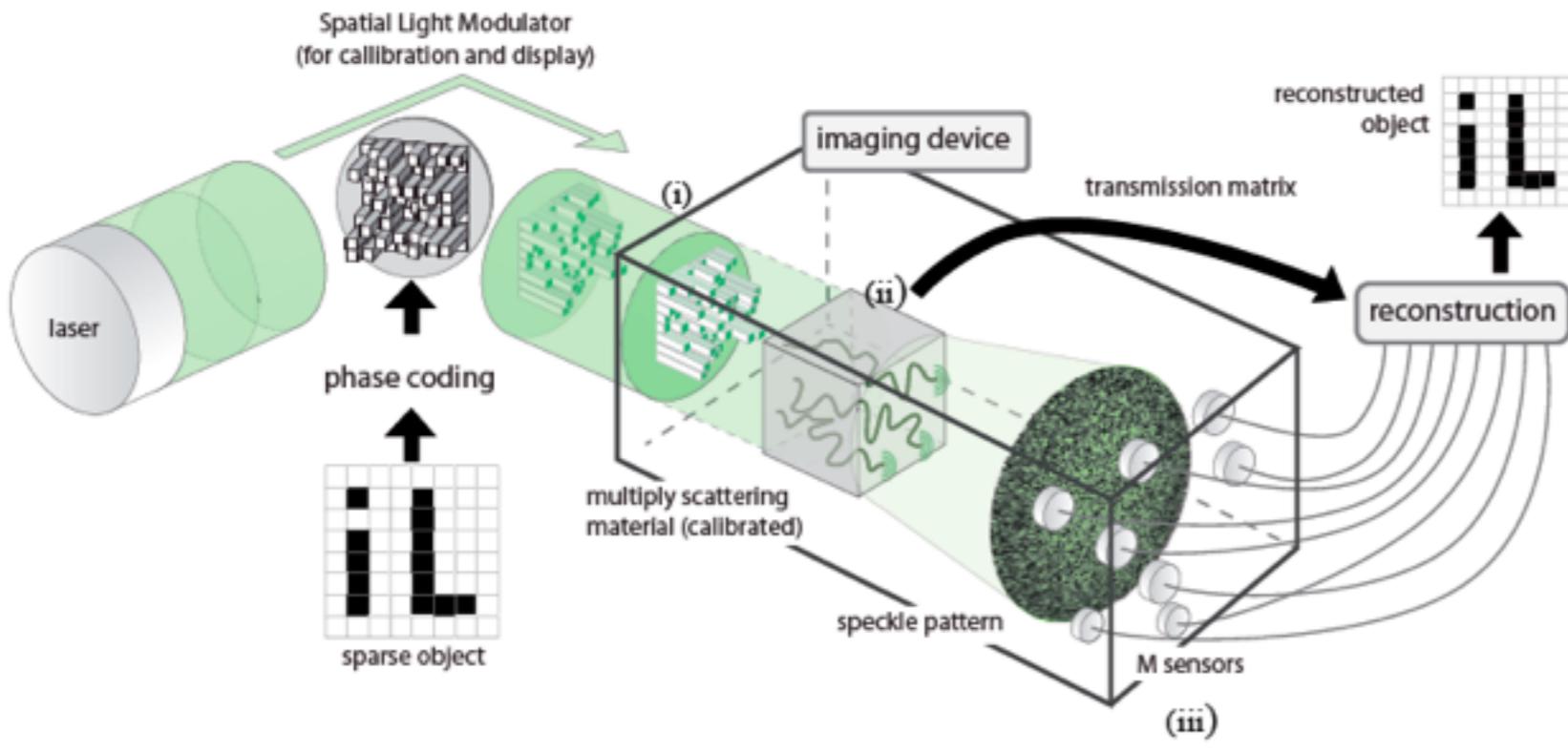


Compressive imaging with scattering media



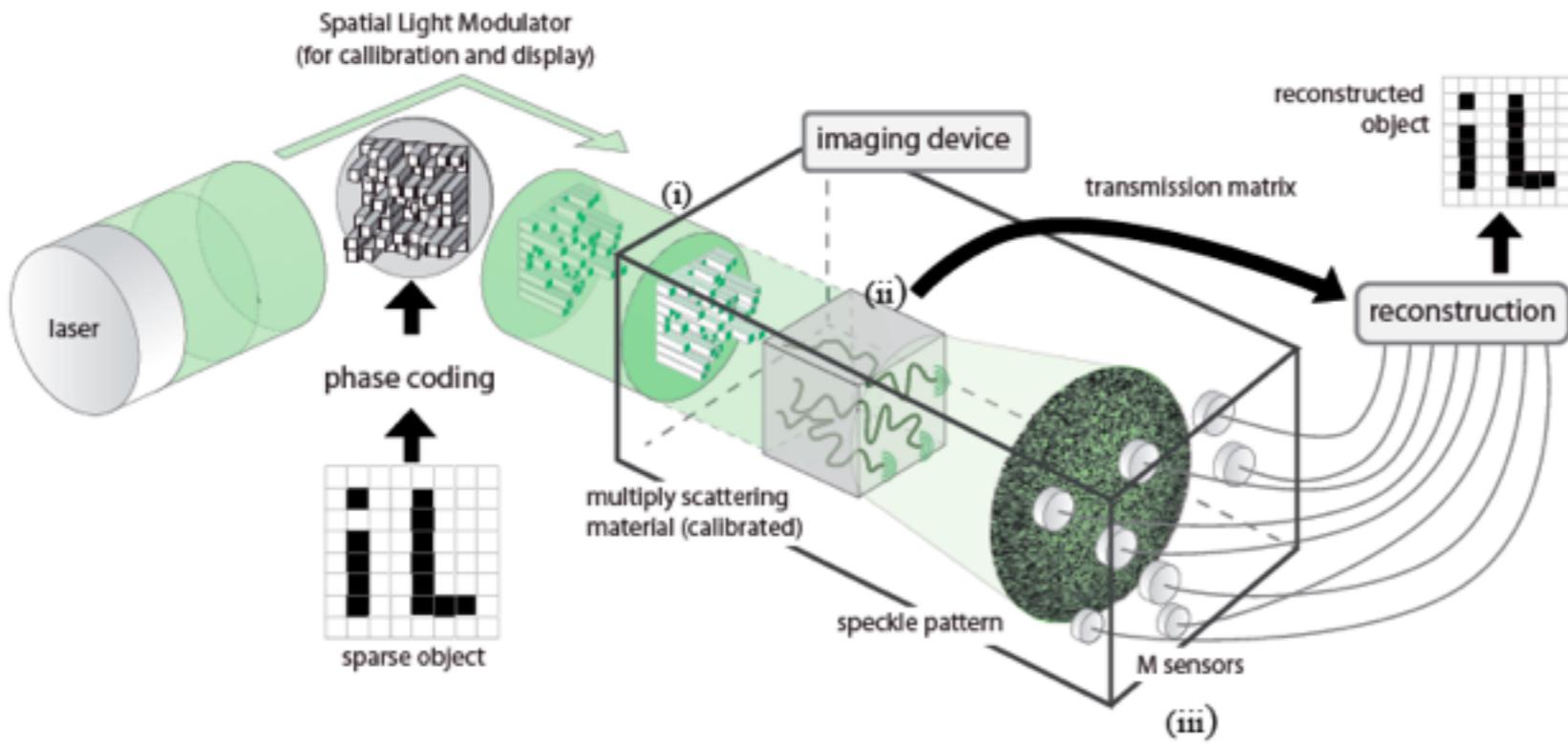
Liutkus et al.,
Scientific reports (4)
(2014)

Compressive imaging with scattering media



Liutkus et al.,
Scientific reports (4)
(2014)

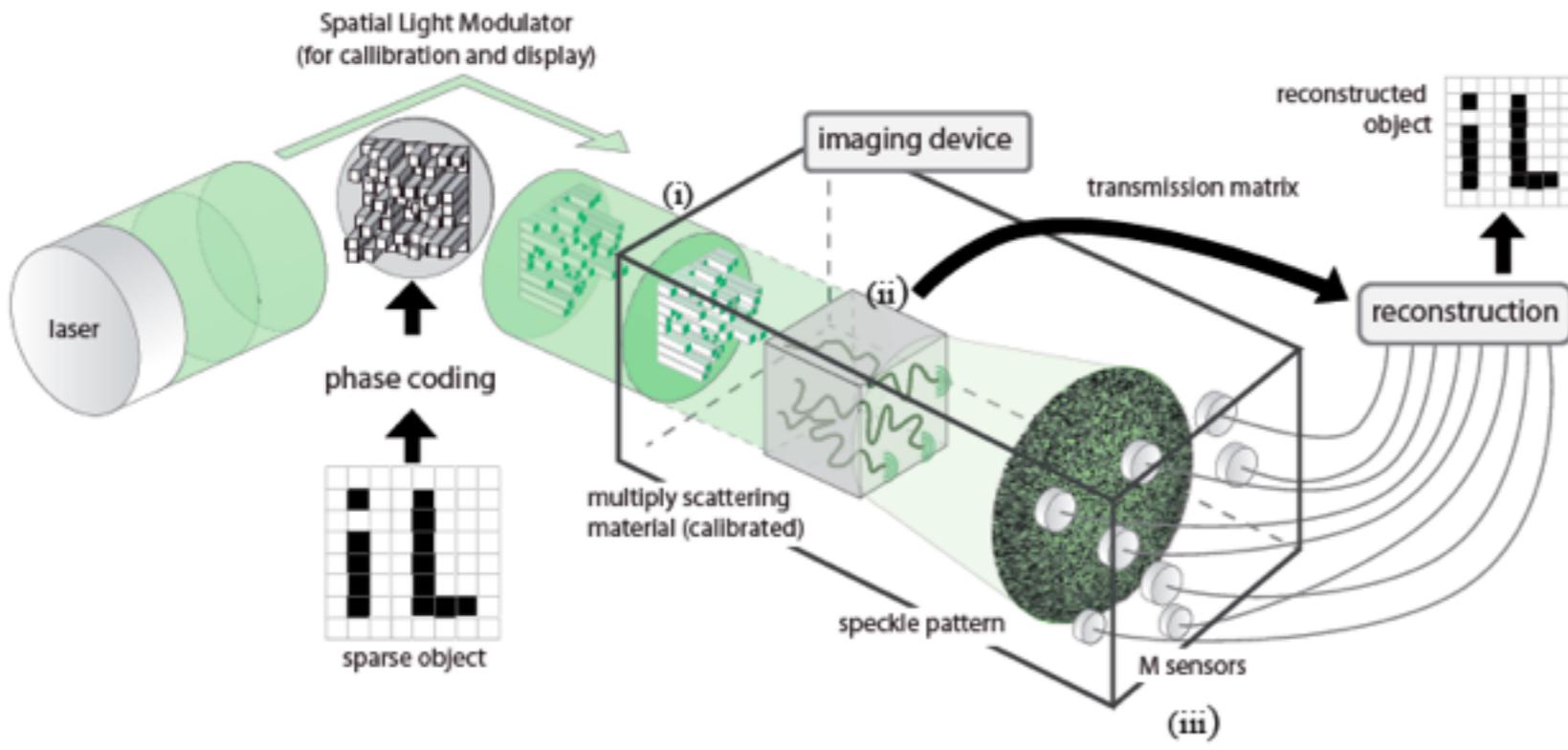
Compressive imaging with scattering media



Liutkus et al.,
Scientific reports (4)
(2014)

- From *imaging through scattering media* (challenge)
to *using scattering media to better image* (opportunity)

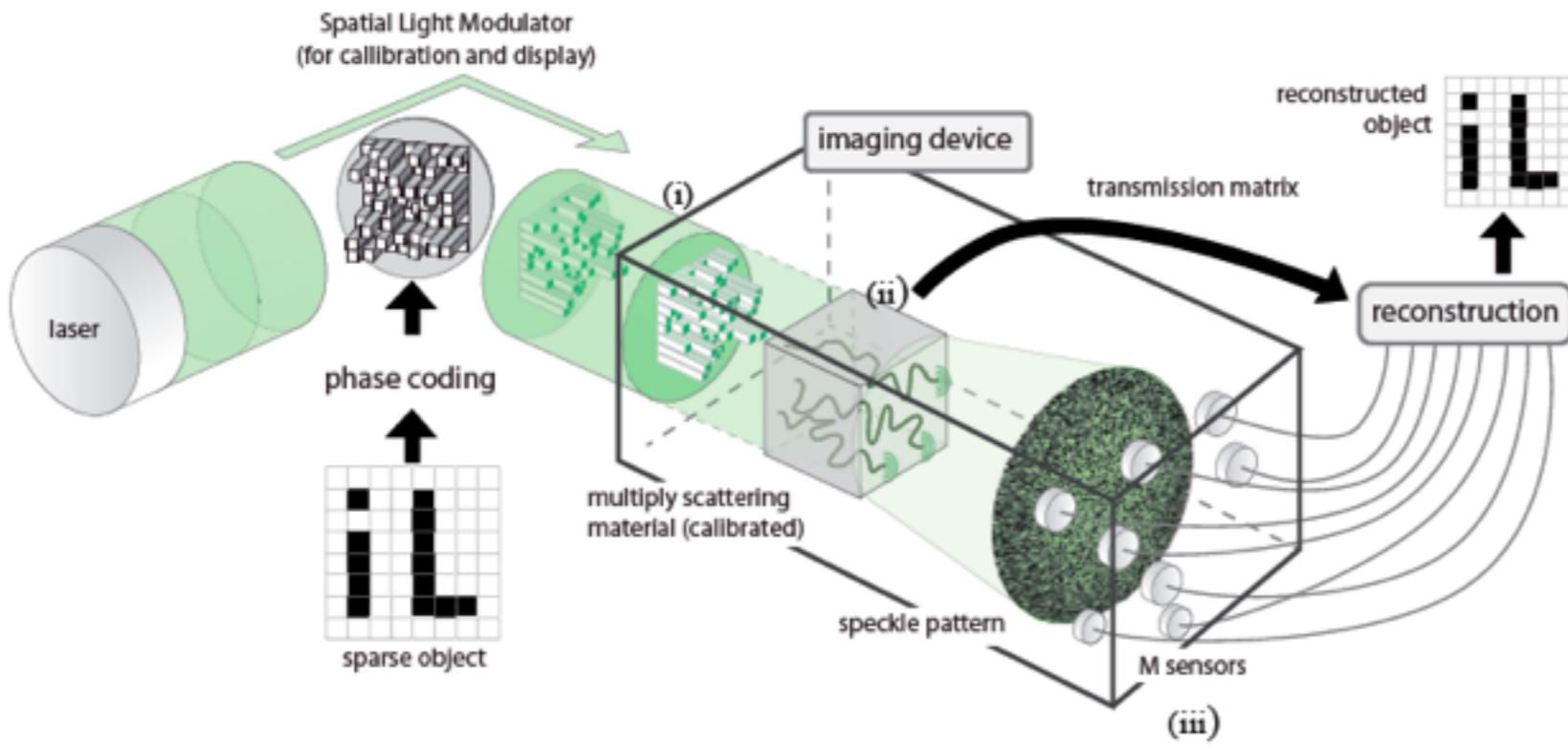
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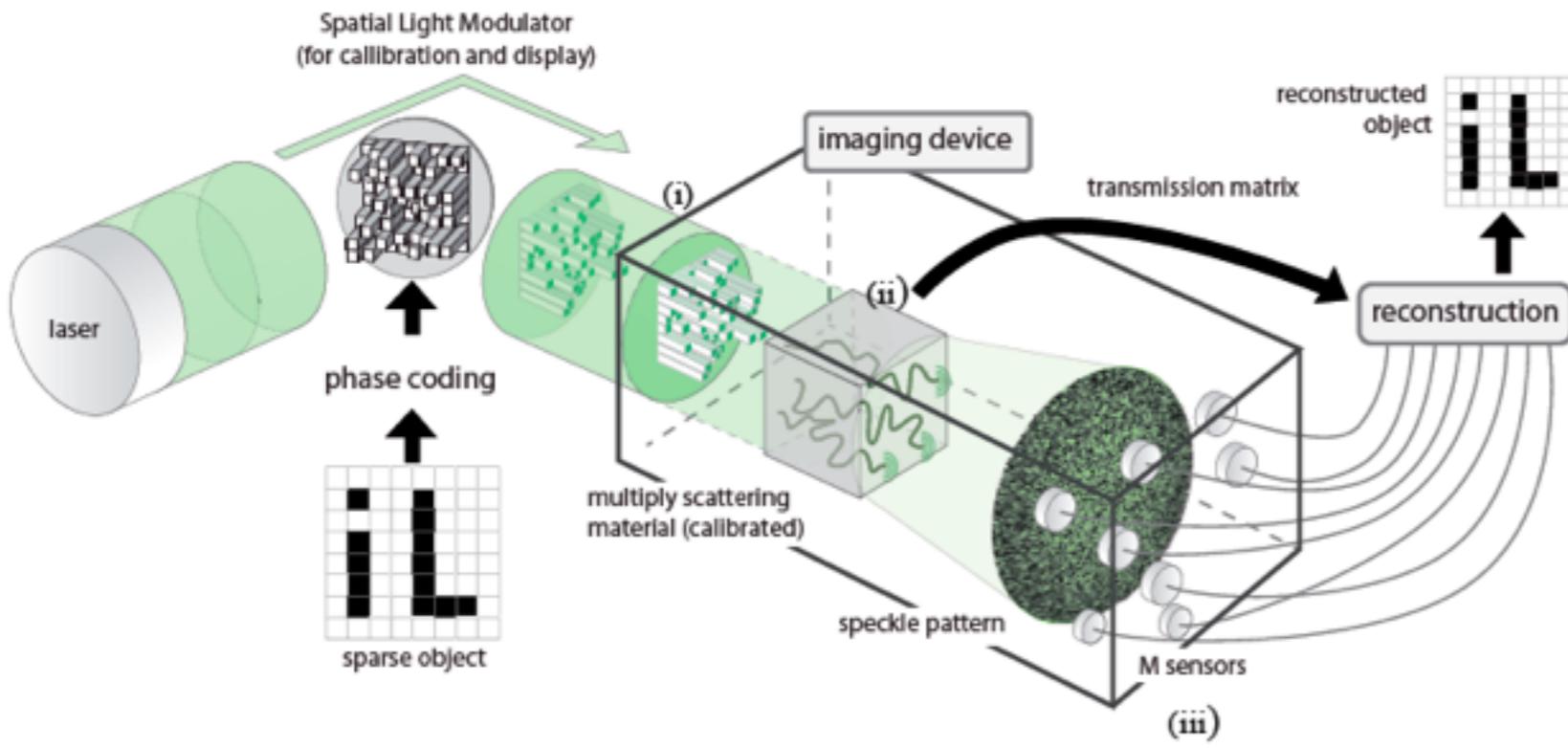
Compressive imaging with scattering media



Liutkus et al.,
Scientific reports (4)
(2014)

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Compressive imaging with scattering media



Liutkus et al.,
Scientific reports (4)
(2014)

- From *imaging through scattering media* (challenge) to *using scattering media to better image* (opportunity)
- Randomness helps ! “optimal” scrambling of information
- Trade hardware for calibration
- A deep connection between *sensing* and *sampling*

Compressively Sensing acoustic fields

- Compressed sensing is a way to put prior information about the signal at the acquisition stage (sparse regularization) and a way to design corresponding efficient measurements

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