



Physics of MRF Regularization for Segmentation of Materials Microstructure Images

Jeff Simmon†
Craig Przybyla†
Stephen Bricker‡
Dae Woo Kim*
Mary Comer*

†Materials and Manufacturing Directorate; Air Force Research Laboratory; OH 45433; USA

‡Department of Electrical and Computer Engineering; University of Dayton; OH 45469; USA

**Department of Electrical and Computer Engineering; Purdue University; IN 47907*



Electronic Imaging for Microscopy-2005 to Date



Incorporate and adapt modern imaging methods for analysis of microscope data

Materials

*Marc De Graef (CMU)
Craig Przybyla,
Lawrence Drummy,
Jeff Simmons (AFRL)*

EE/Comp. Sci

*Charles Bouman, Mary Comer,
Ilya Pollak (Purdue)
Alfred Hero (U. Mich)
Song Wong (U. South Carolina)
Russel Hardie (U. Dayton)*

Bayesian Segmentation

*(EM/MPM)
Comer*

Dictionary-based inversion

*(EBSD)
Hero*

Graphcut Segmentation

*(topology preserving)
Wong*

Anomaly detection

*(automatic classification of EBSD
Irregular features in large datasets)
Hero, Hardie*

Dictionary matching segmentation

*(matching pursuits)
Pollak*

Feature Extraction

*(velocity gradient
moment invariant texture classifications)
Przybyla, De Graef*

Stabilized inverse diffusion (discontinuities)

*(SIDE)
Pollak*

TEM Tomographic reconstruction

*(HAADF STEM, bright field TEM)
Drummy, Bouman*

...always outdated



Opportunity: Co-Evolution of the Ising Model

Physics

Statistical
Mechanics

- **Ising (1924)**
1-D Ising Ferromagnet model
- **Onsager (1944)**
2-D Ising Ferromagnet model
- **Potts (1952)**
Extension to multiple spin states

Hammersley&Clifford(1971)

general method for MRF priors

Besag (1974)

proof of H-C theorem

German and German(1984)

MC/MC MAP est.

Marroquin, et al. (1987)

Gibbs Sampler MPM

Regularization

Statistics/Imaging

Evolution

Binder (1968)

Metropolis M/C-thermodynamics

Liebowitz, et al. (1976)

Ising spin systems

Srolovitz, Rollett,

Holm, et al. (1988)

Evolution of poly-crystalline mat'ls

Miodownik, et al. (2000)

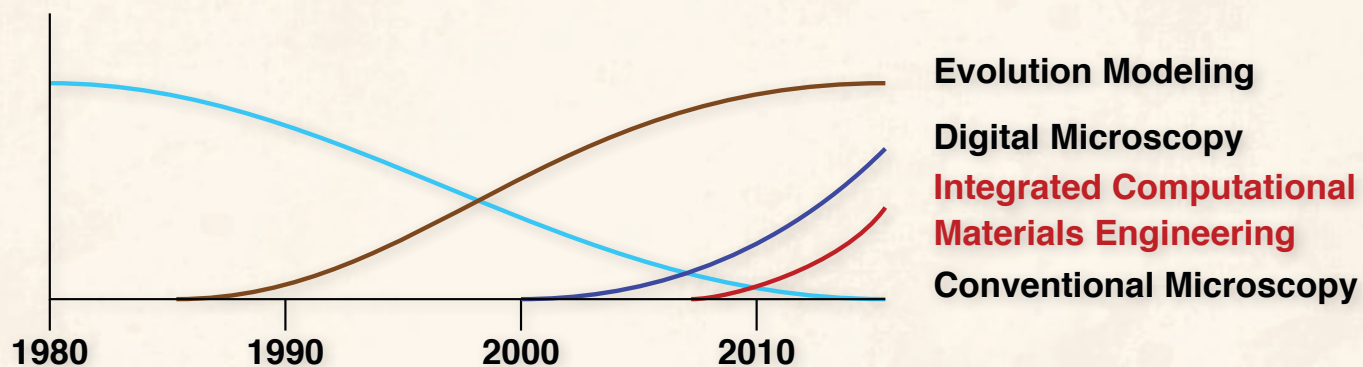
second phase pinning

Materials



Motivation

Research Trends



Opportunity

Fusion of evolution modeling with digital microscopy

parameter estimation for evolution models
physics-based regularization for image analysis

Presentation Goals

Show where MRF regularization \Leftrightarrow real material behavior
Uncover unexploited materials properties implicit in MRF



Outline

Motivation

Integration of techniques

Legacy: evolution models

Emerging: digital microscopy

Methods

EM/MPM regularized segmentation

Physics-dominated extreme

Surface Science

`Energy penalty' \Rightarrow `interfacial energy'

Coarse graining

Physics in MRF Segmentations

Materials physics intrinsic in MRF regularization

Commonly observed phenomena

Qualitative

Potential Developments

Physics not in conventional MRF regularization

Materials specific extensions

Conclusions



Methods

EM/MPM Segmentation

Forward model

mixture of Gaussians

EM algorithm for fitting histogram

Comer and Delp, (2000)

$$f_{Y|X}(y|x, \theta) = \prod_{r=1}^N \frac{1}{\sqrt{2\pi\sigma_{x_r}^2}} \exp\left(-\frac{(y_r - \mu_{x_r})^2}{2\sigma_{x_r}^2}\right)$$

Regularization

4-neighbor MRF

$$p_X(x) = \frac{1}{Z} \exp\left(-\sum_{\{r,s\} \in C} \beta_{x_r, x_s} (1 - \delta_{x_r, x_s})\right)$$

$\delta_{ij} = \text{Kronekar delta}$

$C = 4\text{-neighbor clique}$

Estimation of posterior marginals

Markov chain Monte Carlo

Phantoms

Slight composition gradients + Poisson noise

artifact boundaries: pure physics



Extremes of Regularization

100%
Observation

Regularization

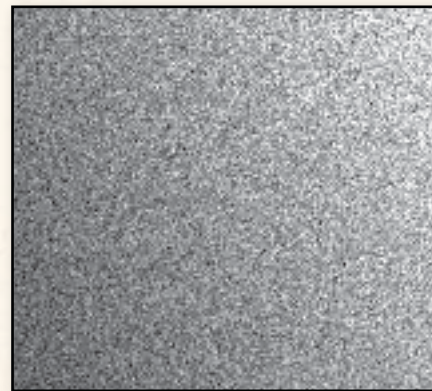
100%
Physics

Strong
Regularization

Large hyperparameter

Low contrast

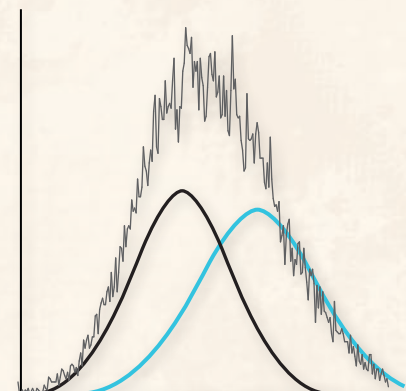
Regularization
Dominated
Segmentation



*Gradient
+ Poisson noise*



Segmentation



histogram model
*Artifact boundary
(mixture of Gaussians model)*



Surface Science 101 1/2: Coarse Graining



Markov Random Field (p_X)

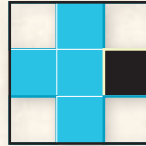
$$1/z \exp \left[-\sum_{ij} \beta^{\alpha\beta} (1 - \delta^{\alpha\beta}_{ij}) \right]$$

Smoothing:

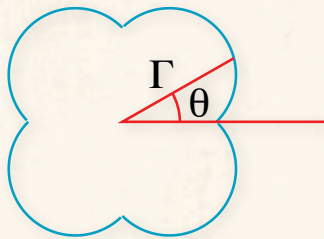
penalty unless both pixels are same class

$\beta^{\alpha\beta}$ spatial interaction parameters

dependent on classes involved

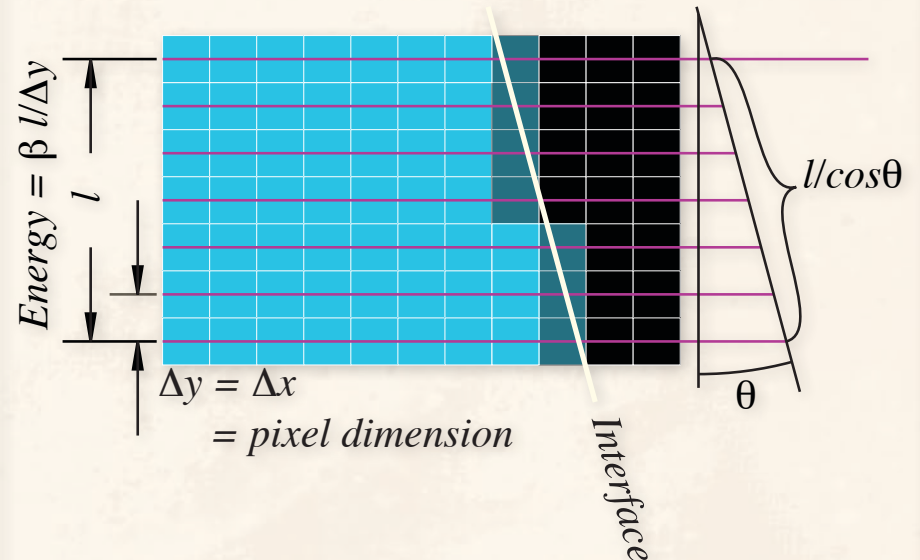


Anisotropic interface energy density



*Wulff plot for 4-neighbor
interfacial energy*

Coarse-Graining: $\beta \rightarrow \Gamma$



Energy density: horizontal

$$(\beta / \Delta y) \cos \theta$$

Energy density: vertical

$$(\beta / \Delta x) \sin \theta$$

Total interfacial energy density

$$\Gamma_{0^\circ K} = \sqrt{2}(\beta / \Delta y) \sin(\theta + \pi/4) \quad \theta \in [0, \pi/2]$$



Surface Science 101 2/2: Interface 'Tension'



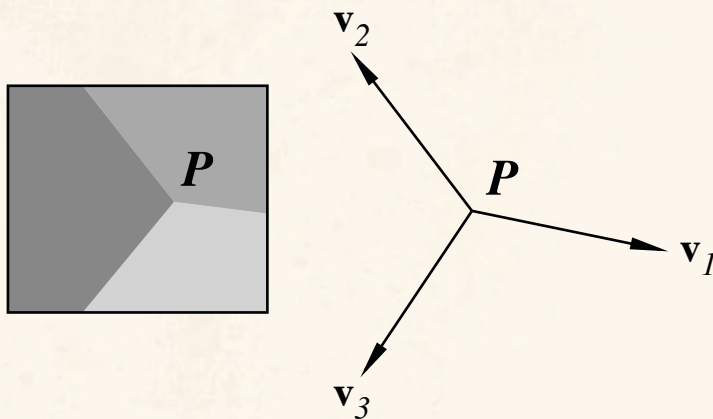
'Energy' \Rightarrow 'force'

Regularizing boundary with 'energy'

$$\Rightarrow E(\mathbf{P})$$

$$\Rightarrow \mathbf{F} = \nabla E(\mathbf{P})$$

$$\text{Equilibrium} \Rightarrow \mathbf{F} = 0$$



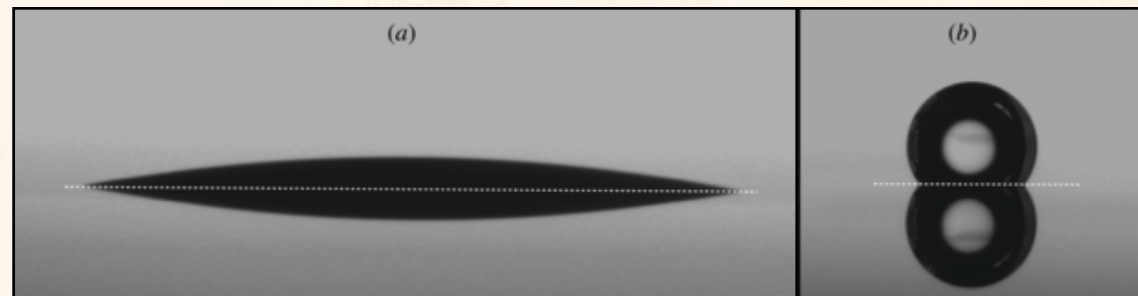
$$\sum_i \Gamma_i \mathbf{v}_i = 0$$

\mathbf{v}_i = vector from P
in direction of boundary

$$\Gamma_i \triangleq \left| \frac{dE_i}{d\mathbf{v}_i} \right|$$

Example

Fig. 6 of
Gari Arutinov, et al.,
J. Micromech. Microeng.,
22, 115022, (2012).



Oxidized SiO₂ substrate

*Gold coated
SiO₂ substrate*



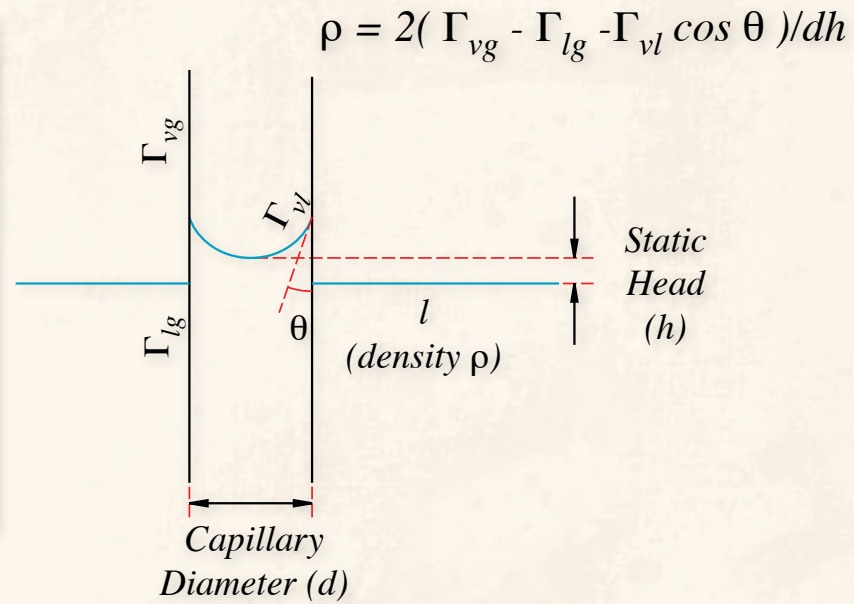
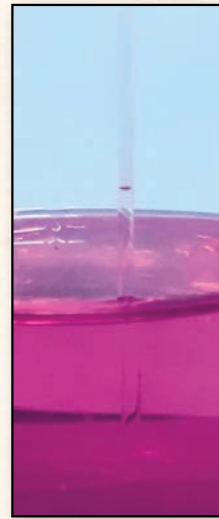
Surf. Sci. in MRF-Regularized Segmentations

Capillarity	Surface energy induced lifting of one phase <i>extending region with interface length penalty</i>
Wetting	Coating phases with `boundary phase' <i>thin region separating two larger regions of different classes</i>
Pinning	Pinning <i>interaction of boundaries with regions</i>

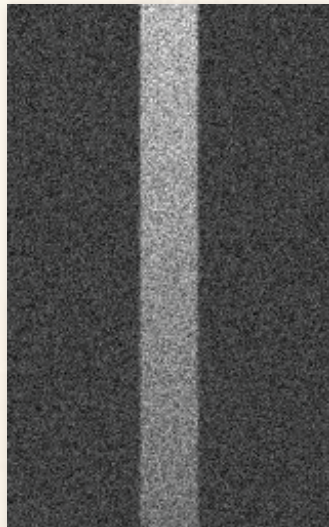


Capillarity Lifting

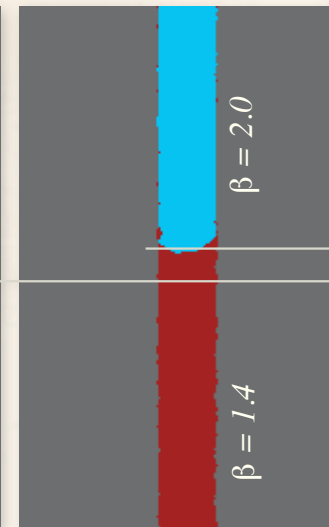
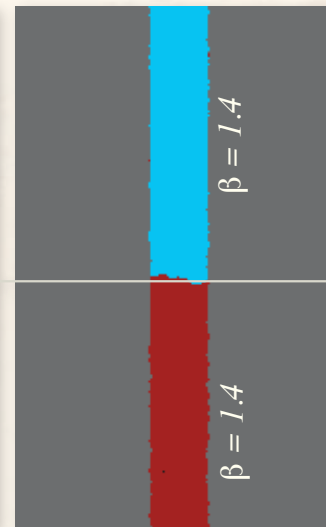
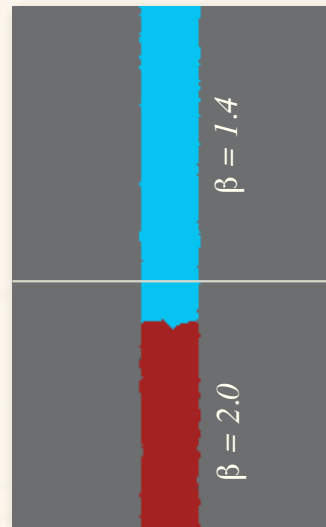
Physics



Regularized Segmentation



Phantom



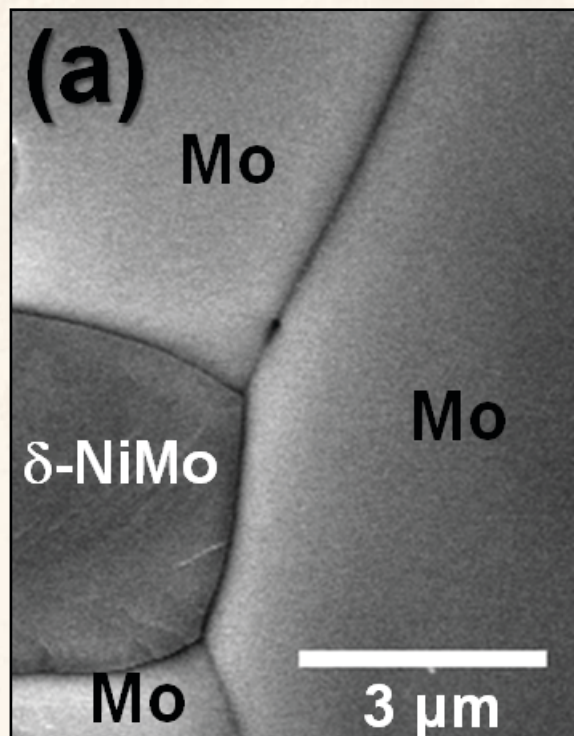
artifact
 static
 head



Wetting

Mo grains at **two different temperatures**

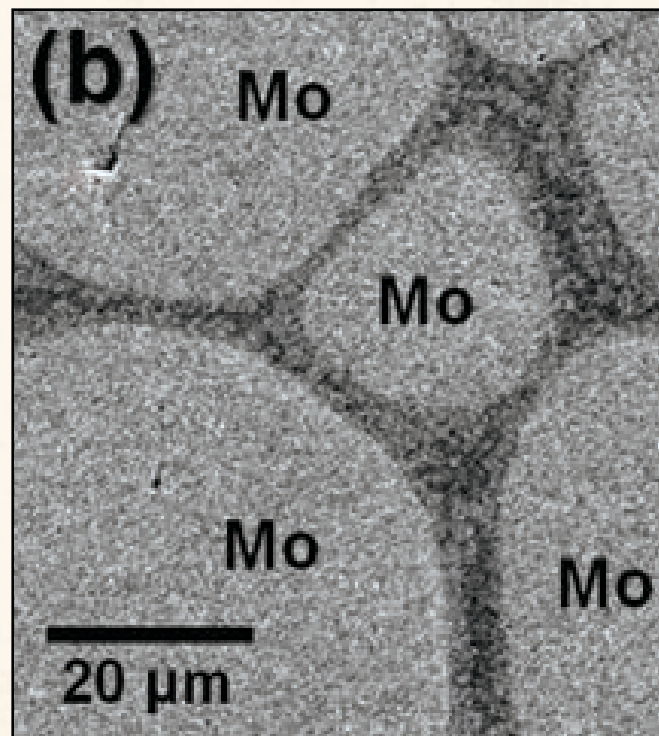
interfacial energy fn of temperature



Non-wet interfaces

SEM image

*Mo-12.4%Ni quenched from **1344C***



Wet interfaces

Cross-sectional SEM image

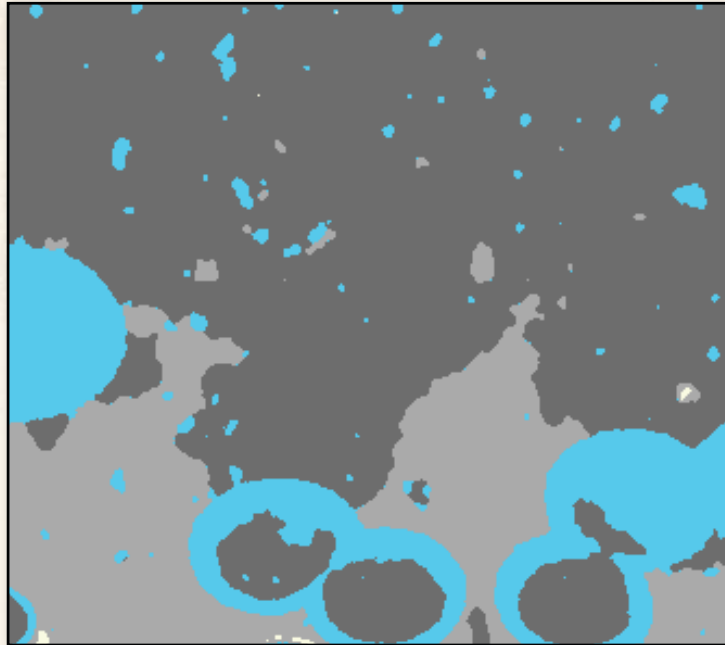
*Mo-12.4%Ni quenched from **1495C***

Courtesy Jian Luo

Xiaomeng Shi and Jian Luo, Appl. Phys. Lett., 94, 251 908, (2009)



Thin Region Separating Two Regions



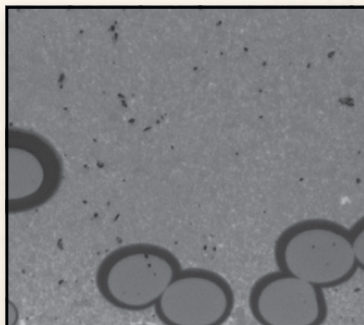
Segmented image

$\beta_{0,1}=0.9, \beta_{0,2}=0.5, \beta_{1,2}=0.9$
non-wet interface



Segmented image

$\beta_{0,1}=0.9, \beta_{0,2}=0.9, \beta_{1,2}=1.8$
wet interface



Raw image

SiC fiber in SiC matrix. BN coating

Imaging:

optical
slight intensity gradient in BG

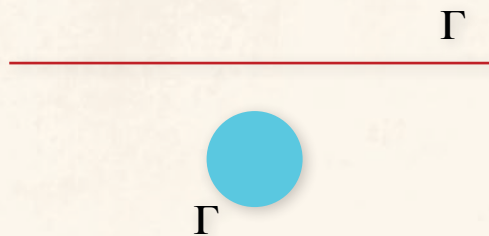


Ex 2: Zener Pinning for Grain Refinement

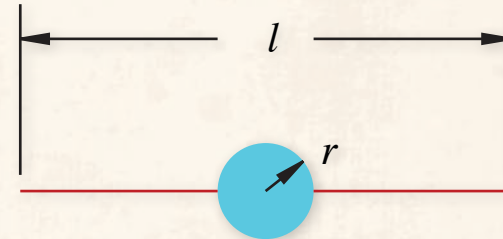
Friction Force

Interface 'attracted to' particles

Reduces boundary penalty



$$E = 2\pi r \Gamma + l \Gamma$$



$$E = 2\pi r \Gamma + l \Gamma - 2r \Gamma$$

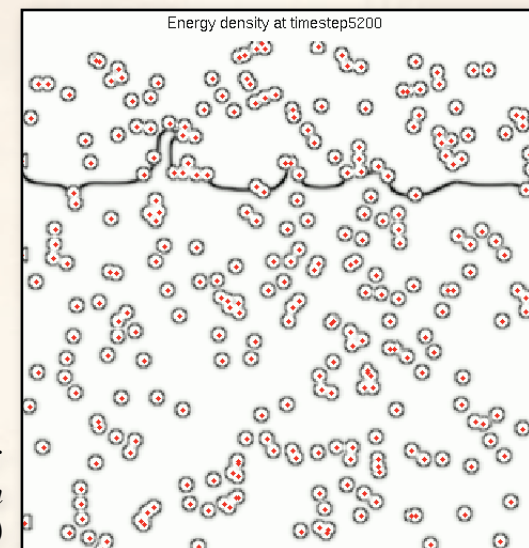
'Friction force' of $2r\Gamma$ pinning the boundary

Zener Pinning Mechanism

*Zener, unpublished
(cited C.S. Smith, 1948)*

Used in alloy design

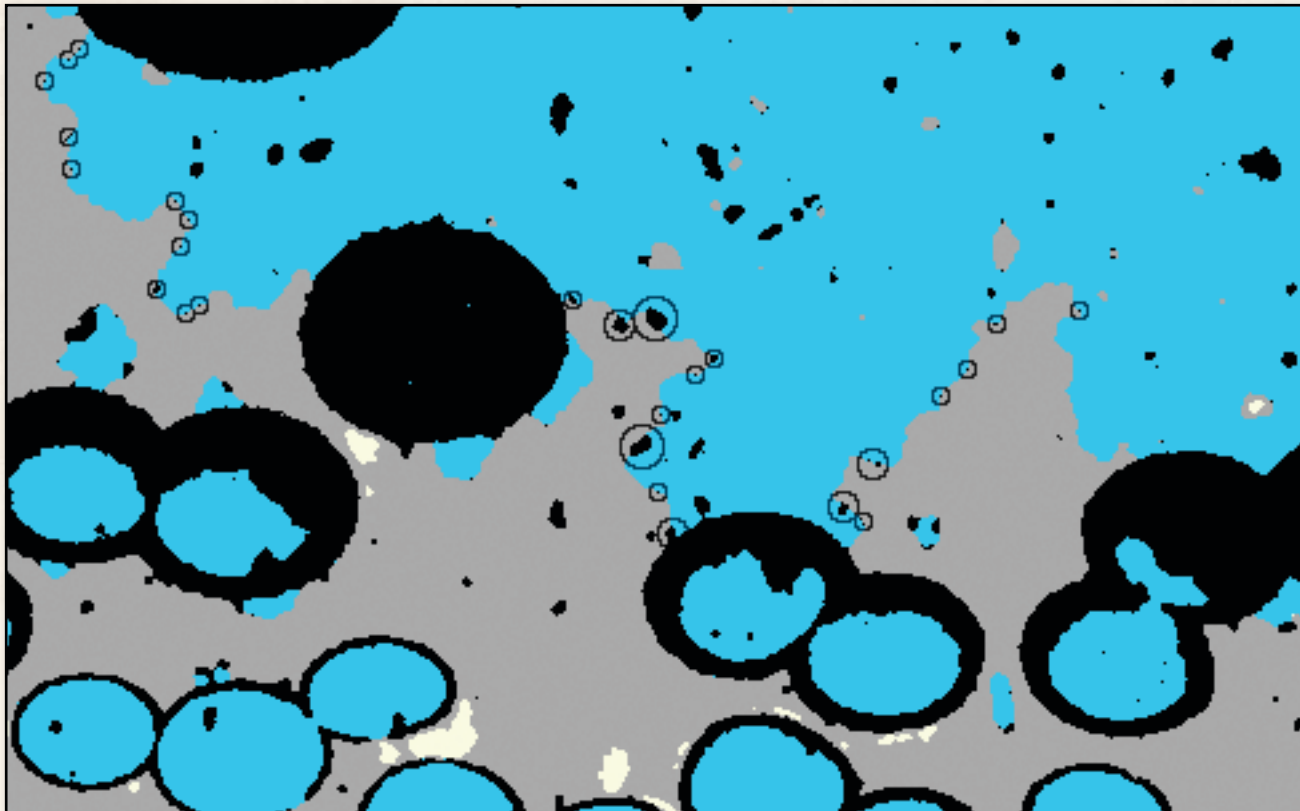
stop grains from growing



Source:
[commons.wikimedia.com](https://commons.wikimedia.com/wiki/File:Zener_Pinning)
(Zener Pinning)



Interaction of Boundaries with Regions



Individual particles in image **'pin' boundary**

Boundary intersects large black classes at **right angles**

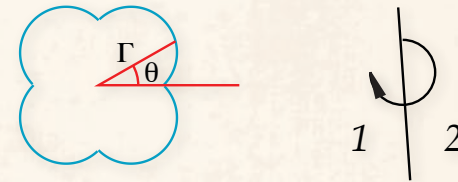


Future Materials-Specific Extensions

Anisotropic interfacial energy

cusps imply torques

8-neighbor has cusps in [1 1] directions



Coarse graining: **quantitative link to materials**

Γ can be estimated from β with Monte Carlo (Binder)

Requires a 'temperature' of the MRF

Thermodynamics

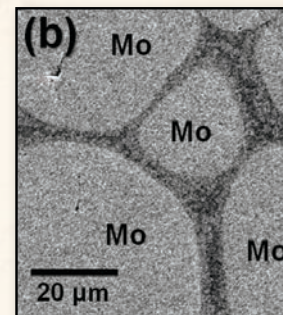
$$F = E - TH$$

Statistical Mechanics

$$T = \frac{RMS(E)}{k_b N c_v}$$

Wetting: **potential robust segmentation**

experiment: boundaries between same classes





Conclusions

MRF is a generalization of Ising Model

solid state physics

Reproduces classical surface science

qualitative

Materials-specific extensions possible

Ising model reflects actual materials behavior

Expected uses

inpainting

boundary orientations

quantitative regularization

separation of spatially close regions in segmentations