Unary adaptive subtraction of joint multiple models with complex wavelet frames

Sergi Ventosa, Institut de Physique du Globe de Paris; Sylvain Le Roy, Irène Huard, Antonio Pica, CGGVeritas; Hérald Rabeson, Laurent Duval, IFP Energies nouvelles*

SUMMARY

Multiple attenuation is one of the greatest challenges in seismic processing. Due to the high cross-correlation between primaries and multiples, attenuating the latter without distorting the former is a complicated problem. We propose here a joint multiple model-based adaptive subtraction, using single-sample unary filters' estimation in a complex wavelet transformed domain. The method offers more robustness to incoherent noise through redundant decomposition. It is first tested on synthetic data, then applied on real-field data, with a single-model adaptation and a combination of several multiple models.

INTRODUCTION

Multiples correspond to unwanted coherent events related to wavefield reflection bounces on given surfaces. We refer to Verschuur (2006) for a precise typology. Data can be transformed to an appropriate domain, to reduce overlap between primaries and multiples. After multiple suppression, the filtered data is mapped back to data domain. Alternatively, prediction filters (Spitz et al., 2009), and variations thereof, have demonstrated excellent performance. Recently, modeling based techniques, such as surface related multiple estimation (SRME), allow data- or model-driven multiple removal (Weisser et al., 2006; Lin et al., 2004). These methods, based on multiple models, consist in predicting then subtracting multiple events from original seismic data.

Since primaries and multiples are not orthogonal, hybrid methods mix both transform and prediction approaches. A recent trend focuses on wavelet-related approaches, with a review in **?**. They better promote sparsity (Lin and Herrmann, 2011) in exploiting slight differences between primaries and multiples. For instance, Ahmed (2007) perform matching in a discrete wavelet domain, while Donno et al. (2010); Neelamani et al. (2010) use (complex) curvelets.

The present work builds upon Ventosa et al. (2011, 2012). We combine sparsifying transforms and their associated matched filters via 1) non-stationary Wiener matching filters, 2) complex trace processing, and 3) continuous wavelet frames.

The complex Morlet wavelet frame emulates complex derivatives computed at specific scales. The adaptation in the wavelet domain is performed at each scale with a "unary" filter, e.g., a one-coefficient complex matching operator, accounting for localized phase and amplitude variations between data and models. The flexible redundancy and the "complex trace" effect of this wavelet frame allow a better management of time variability in model misalignment errors.

We address two specific issues. Firstly, it is well known that redundancy in a transformation may improve the robustness to noises. However, it hampers the overall algorithmic efficiency. Secondly, when two or more multiple models are available (Mei and Zou, 2010), locally combining multiple models with varying weight improves upon separate processing and averaging procedures.

The methodology is two-fold: firstly choose an appropriate redundancy in the transform to ensure sufficient robustness to incoherent disturbances. It is performed on a synthetic dataset with varying redundancy and noise levels. Secondly, the single-model complex wavelet adaptive multiple subtraction from Ventosa et al. (2012) is extended to a joint multi-model approach. The benefits of the proposed methods are demonstrated on field data with three different multiple models.

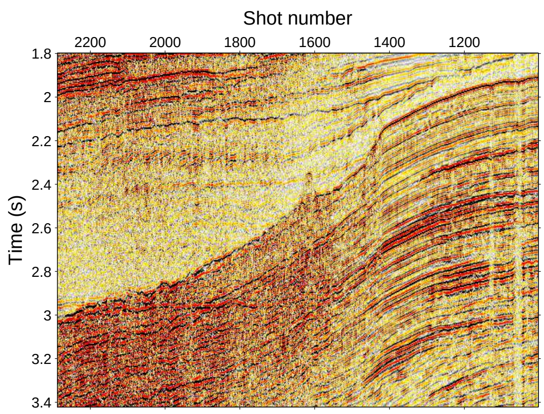

Figure 1: Portion of first receiver plane: raw data.

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WAVELET-DOMAIN MULTIPLE SUBTRACTION

A classical trace observation model is:

$$d[n] = p[n] + m[n] + w[n]\,, \tag{1}$$

where $d[n]$, $p[n]$, $m[n]$ and $w[n]$ denote the recorded data, primary events, multiples and background noise, respectively, at discrete time index n.

Complex wavelet transform decomposition

We perform a time-scale decomposition of each data $d[n]$ and multiple model $x_k[n]$ traces with a discrete approximation to a continuous wavelet frame. We choose the complex Morlet wavelet since it yields a simple interpretation of amplitude and phase delay in the transformed domain. As it is approximately analytic, it mimics the complex trace, applied to wavelet scales. It writes:

$$\psi(t) = \pi^{-1/4} e^{-i\omega_0 t} e^{-t^2/2}\,, \tag{2}$$

where ω_0 is the central frequency of the modulated Gaussian, and t the continuous time variable.

The associated discrete family of functions is defined as a sampling of the mother wavelet:

$$\psi^{v}_{r,j}[n] = \frac{1}{\sqrt{2^{j+v/V}}}\,\psi\left(\frac{nT - r2^j b_0}{2^{j+v/V}}\right)\,, \tag{3}$$

with T the sampling rate and V the number of voices per octave. Indices $r, j \in \mathbb{Z}$ and $v \in [0, \ldots, V-1]$ denote, respectively, discretized time, octave, and voice. Finally, b_0 stands for the sampling period at scale zero. The overall redundancy, approximately of $2V/b_0$, controls the balance between computational efficiency and robustness.

The time-scale representation of trace $d[n]$ is given by the inner product:

$$\mathbf{d} = d^{v}_{r,j} = \left\langle d[n], \psi^{v}_{r,j}[n] \right\rangle = \sum_n d[n]\overline{\psi^{v}_{r,j}[n]}\,. \tag{4}$$

and written in bold.

Suppose $\hat{\mathbf{d}}$ results from some time-scale processing of $\mathbf{d}$, here model matching and subtraction. Then the resulting filtered trace is synthesized back to the time domain with the dual frame:

$$\hat{d}[n] = \sum_r \sum_{j,v} \hat{d}^{v}_{r,j}\,\widetilde{\psi}^{v}_{r,j}[n]\,, \tag{5}$$

as a sum of the dual frame components, $\widetilde{\psi}^{v}_{r,j}[n]$, weighted by the adapted multiple decomposition, $\hat{d}^{v}_{r,j}$. In practice, the dual synthesis frame is well approximated by the analysis frame ψ up to a constant factor, provided $V \geq 3$ and $b_0 \leq 1.5$.

The discriminative power of the wavelet frame simplifies the reformulation of a long matching-filter design with a combined global and local complex unary filters, minimizing the error between multiple events and their matched model. The straightforward delay estimation allowed by complex Morlet wavelets, together with frame redundancy, drastically reduce reconstruction artifacts, sometimes observed with standard, or orthogonal, discrete wavelet processing (Yu and Whitcombe, 2009).

Single-model unary filter estimation

When the delay difference between model and multiple sequences is less than half a period at all scales, a single multiple model $\mathbf{x}_1$ can be rectified in time-scale, following a least-square-error (LSE) approach. The optimum unary filter, at a given wavelet scale, either in local or global portions of the trace, is defined as the complex scalar a_1 which, multiplied by the time-scale decomposed multiple model $\mathbf{x}_1$, makes the filtered dataset orthogonal to the filtered model:

$$a_{\text{opt}} = \underset{a_1}{\arg\min}\, \|\mathbf{d} - a_1\mathbf{x}_1\|^2\,, \tag{6}$$

where the complex scalar a_{opt} compensates local delay and amplitude mismatches. We refer to Ventosa et al. (2012) for additional information.

Joint multiple model unary filter estimation

When several different multiple models $\mathbf{x}_k$ are available, different delays and amplitudes may affect the coupling between each available model and the actual multiple sequence. The above criterion is modified accordingly:

$$\mathbf{a}_{\text{opt}} = \underset{\{a_k\}(k\in K)}{\arg\min} \left\| \mathbf{d} - \sum_k a_k \mathbf{x}_k \right\|^2\,. \tag{7}$$

The optimum value in LSE sense makes the filtered signal $\hat{\mathbf{d}} = \mathbf{d} - \sum_k a_k\mathbf{x}_k$ orthogonal to the k filtered models $a_k\mathbf{x}_k$, which using the inner product is:

$$\left\langle \mathbf{d} - \sum_k a_k\mathbf{x}_k, a_m\mathbf{x}_m \right\rangle = 0 \quad \forall a_m \neq 0\,. \tag{8}$$

Applying linearity on the first argument and conjugate linearity on the second one, we obtain:

$$\langle \mathbf{d}, \mathbf{x}_m \rangle = \sum_k a_k \langle \mathbf{x}_k, \mathbf{x}_m \rangle\,, \tag{9}$$

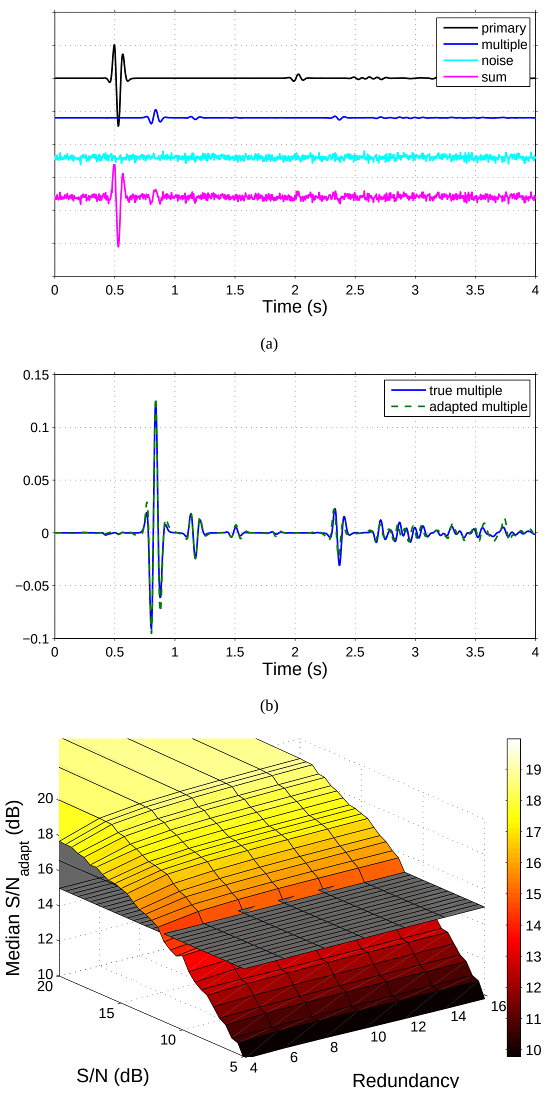

Figure 2: Synthetics signals used for sensitivity analysis. (a) From top to bottom: primary, multiple model, random noise (S/N of 5 dB), sum.); (b) True model and adapted model after 1D unary filter adaptation; (c) Sensitivity analysis to random noise and redundancy levels: median values of S/N (adapted model vs true model) computed on 100 random noise realizations at each point.

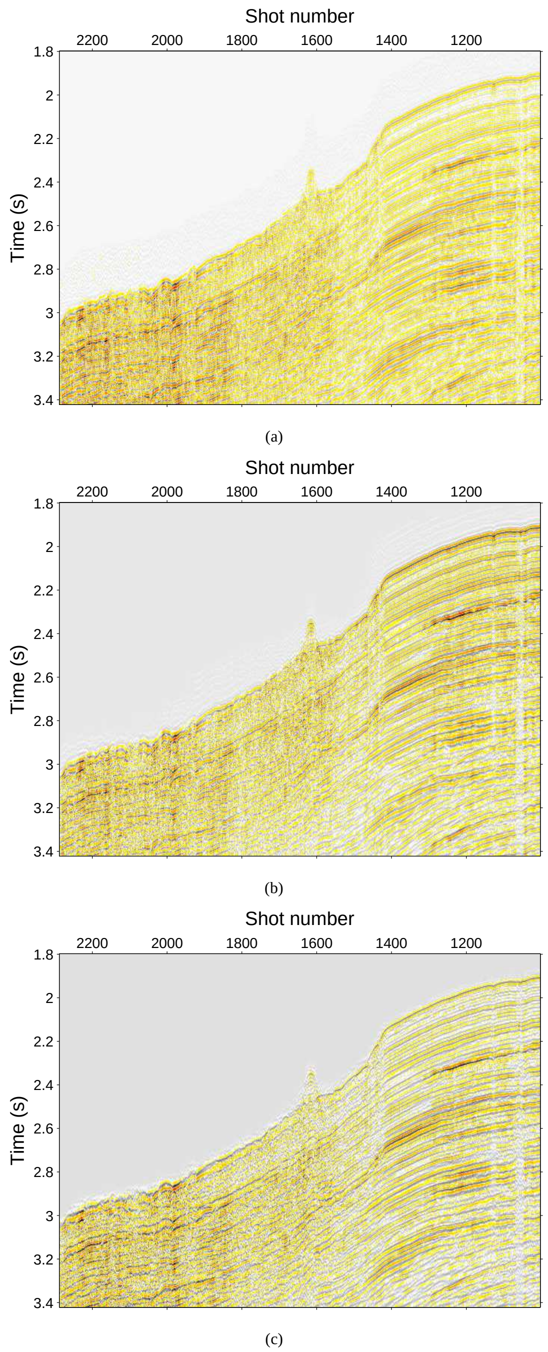

Figure 3: Models based on (a) wave equation (b) convolution (c) and parabolic Radon.

that is, the vector Wiener equations for complex signals. In practice, since some of the multiple models are locally similar, the cross-correlation matrix is frequently close to singular. The solution is obtained by keeping eigenvectors with corresponding eigenvalues above a prescribed threshold.

RESULTS

Noise robustness and redundancy assessment

The proposed algorithm offers some flexibility, such as selection of redundancy in the wavelet decomposition. The quality of adaptation is evaluated using synthetic signals (Figures 2a, 2b) by varying random noise level (from 5 dB to 20 dB with 0.5 dB steps) and redundancy (from 4 to 16 with steps of 2). In the Monte-Carlo approach, 100 realizations of white Gaussian noise were generated for each point. Figure 2c shows corresponding results with $b_0 = 1$. The resulting signal-to-noise (S/N) ratio is estimated from the true m and the adapted model m_{adapt}, using the following formula:

$$\mathrm{S/N}_{\text{adapt}} = 10\log_{10}\left(\frac{||m||^2}{||m-m_{\text{adapt}}||^2}\right). \qquad (10)$$

Model adaptation is more robust to random noise thanks to redundancy. The improvement in S/N is negligible for large redundancy. Thus, we choose a redundancy of 8 to process field data, this value offering a good compromise between adaptation quality and computational time.

Joint multiple models field dataset

Figure 1 represents a part of the first common receiver plane from a 3D read marine dataset. Several models (Figure 3) were obtained with wave equation modeling, convolution (3D-SRME) and parabolic Radon. The subtraction results (Figure 4) represent the filtered data after using the unary complex filters: with the 1D unary complex filter approach. A single model already yields an efficient multiple attenuation; however, the algorithm extension to a joint adaptation of several multiple models allows to take into account a more diverse multiple information and provides better attenuation. For instance, some multiples seem to be better attenuated around 3s, thanks to the joint multiple model approach.

CONCLUSION

We propose a joint multiple model-based adaptive subtraction which combines complex Morlet wavelet frame with unary complex Wiener filters. The flexible redundancy in the wavelet frame implementation allows the design of fast unary filters, providing an elegant, and computationally efficient, non-stationary joint multiple model adaptation. This redundancy, chosen with simple test on artificially degraded data, additionally yield robustness to incoherent noises. The computational efficiency of the proposed algorithm allows for reduced memory footprint and higher code parallelization.

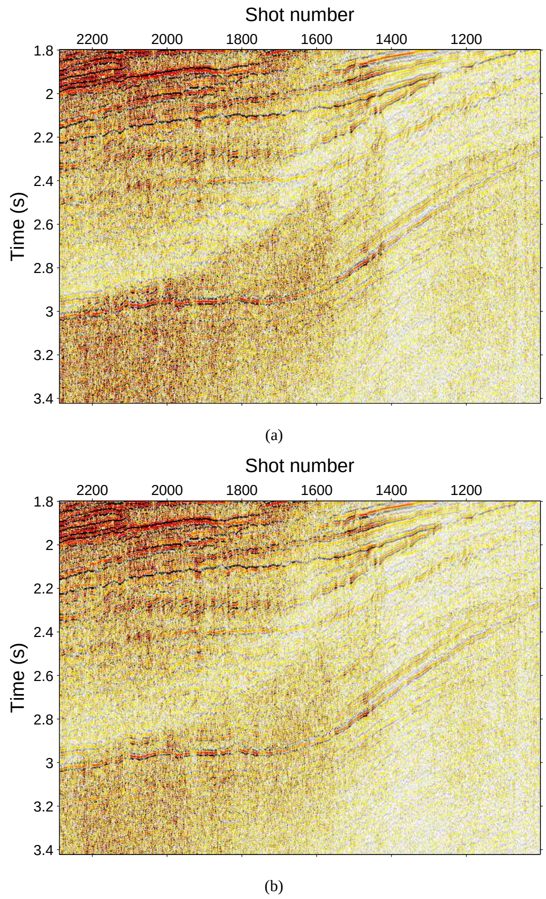

Figure 4: Subtraction results using complex wavelet unary filters with (a) parabolic Radon, (b) joint models.

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