Complex wavelet adaptive multiple subtraction with unary filters

Sergi Ventosa, Herald Rabeson, Patrice Ricarte and Laurent Duval
Introduction

- Multiple contamination is one of the greatest challenges in seismic processing (Backus, 1959; Verschuur and Berkhout, 1992; Matson and Dragoset, 2005)

- Multiple recognition - Characteristics
  - Periodicity
  - MoveOut (Velocity and curbature)

- Multiple attenuation methods
  - Filtering methods (Kelamis et al, 2008)
    - Relay on differentiating features
  - Predictive suppression methods (Pica et al, 2005; Dragoset et al, 2010)
    - Relay on prior knowledge to build a multiple model
Introduction: Primaries & Multiples

- Original data at near receiver plane (non-stacked)
Introduction: Multiple model

- The model is not accurate enough for a plain subtraction.
Introduction: Data & Multiple model

- A piece of the 100\textsuperscript{th} trace

- The multiple prediction method has limitations that lead to imperfect multiple models.
  - An adaptive subtraction algorithm is needed.
Introduction: Adaptive subtraction

- **Main challenges:**
  - Primaries and multiple are not fully uncorrelated, as they are generated from the same source.
  - The variations on amplitude, waveform and delay impose strong constraints on the minimum filter length.

- **Standard approaches:**
  - Minimum $l_2$-norm:
    - A long global filter to compensate systematic differences.
    - A short local filter to compensate the differences that remains.

- **Other approaches:**
  - Minimum $l_1$-norm (Guitton and Verschuur, 2004)
  - Work in a transformed domain.
Outline

- Introduction
- Complex wavelet adaptive multiple subtraction with unary filters
  - CWT: Implementation
  - Amplitude and phase estimation
  - Integer delay estimation
- Adaptive subtraction algorithm results
Complex wavelet adaptive unary filters

- **Main objective**
  - Decompose a complicate wide-band problem into a set of more tractable narrow-band problems.

- **Main properties**
  - Controlled redundancy with frames of wavelets.
  - Simplifies the filter design:
    - Enables the reduction of the filter length up to a single sample.
  - Increase the adaptation capability.
Complex wavelet adaptive unary filters

- CWT implementation using frames of wavelet
  - Family of functions
    \[ \psi_{r,j,v}[n] = \frac{1}{\sqrt{2^{j+v/V}}} \psi \left( \frac{nT - 2^j b_0}{2^{j+v/V}} \right) \]
  - Frames of wavelets transform
    \[ W d_{j,v}[r] = \langle d[n], \psi_{r,j,v}[n] \rangle = \sum_n d[n] \psi_{r,j,v}^*[n] \]
  - Complex Morlet Wavelet
    \[ \psi(t) = \pi^{-1/4} e^{-j\omega_0 t} e^{-t^2/2} \]
    \[ \omega_0 = \pi \sqrt{2/\ln(2)} \]
    \[ Q = 2.27 \]
Complex wavelet adaptive unary filters

CWT: Implementation

**Data and Model**

- **Main parameters of the CWT**
  - Central freq. of the Morlet wavelet: $2\pi$ (Q = 2.7)
  - Mid redundancy, 4 voices/octave + complex (8 times the DWT).
CWT: Example

Data

Model
Amplitude and phase estimation

Main assumptions:
- Small delay (less than the half of the period)
- Minimum energy approach

Problem to solve for each sample in time-scale
- Find the optimum value that multiplied with the model minimize the square mean error with the data

Solution
- Optimum unary Wiener filter for complex signals

\[
a_{opt} = \arg \min_a \xi(a) = \arg \min_a \|d - ax\|^2
\]

\[
a_{opt} = \frac{\langle d, x \rangle}{\|x\|^2}
\]
Integer delay estimation

- Main challenge
  - What can we do when the delay is higher than the half of period of the central frequency?

- One solution
  - Design an unary complex filter with an integer delay.

\[
\xi(a, l) = \sum_r |W_{d,j,v}[r] - a_{j,v}W_{x,j,v}[r - l]|^2 = \|d - a x_l\|^2
\]

\[
a_{opt}[l] = \frac{\langle d, x_l \rangle}{\|x_l\|^2}
\]

- Problem to solve:
  - Find a criterion to select the optimum delay well adapted to the nature of the seismic signals
Integer delay estimation

- Criteria to select the optimum integer delay:
  - Option 1: Minimum mean square error
    \[ l_{opt} = \arg \min_l \xi(a_{opt}[l]) \]
  - Option 2: Maximum normalized crosscorrelation (coherence)
    - Give importance to the waveform over the amplitude
    \[ l_{opt} = \arg \max_l \text{Re} \left[ \frac{\langle d, a_{opt}[l] x_l \rangle}{\|d\| \|a_{opt}[l] x_l\|} \right] \]
Integer delay estimation

Maximum coherence selection method with 0.636 s long rectangular windows
Integer delay estimation

Fractional delay (ms)

Adapted model
Integer delay estimation

Model

Adapted model
Integer delay estimation

Data

Adapted model
Integer delay estimation

Data

Filtered data in time-scale
Integer delay estimation

Filtered data in time-scale

Filtered data in time
Integer delay estimation

Data & original model

Data & adapted model
Subtraction algorithm results: Original data

- Near receiver plane.
Subtraction algorithm results: Multiple model

- The model is not accurate enough for a plain subtraction.
Subtraction algorithm results: Receiver plane

- Adapted model with the 1-D adaptive unary filter.
Subtraction algorithm results: Receiver plane

- Adapted model with the standard 2-D adaptive filter
Subtraction algorithm results: Receiver plane

- Results with the 1-D adaptive unary filter in time-scale.
Subtraction algorithm results: Receiver plane

- Results with the standard 2-D adaptive filter
Subtraction algorithm results: Receiver plane

- Difference
Subtraction algorithm results: Shot plane

- Original data
- 1-D adaptive unary
- Standard 2-D adaptive
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