Introduction

Reducing multiple contamination (Verschuur and Berkhout, 1992; Matson and Dragoset, 2005) represents one of the greatest challenges in seismic processing. Two major aspects differentiate multiples and primary reflections: (1) the velocity of primaries is greater than that of multiples, and (2) multiples are periodic events in contrast with primaries. We can hence classify multiple attenuation methods into two broad categories: *filtering methods* that find a differentiating feature in the primary and the multiple (Kelamis et al., 2008); and *predictive suppression methods* that first predict and then subtract the multiple events from original seismic data (Pica et al., 2005; Weisser et al., 2006). Alas, no single approach fits all scenarios. Most contractors thus propose an extensive portfolio of demultiple algorithms that, in practice, may be combined and cascaded to obtain acceptable solutions.

Predictive multiple suppression methods consist of two main elements: prediction and subtraction. Prediction builds a multiple estimation from the primaries using prior knowledge. Subtraction minimizes amplitude difference and small misalignment between actual multiple events and their predicted models, to maximize multiple attenuation in the input dataset. The efficiency of this suppression strongly depends on the adaptation capability of the matching filter employed in the subtraction element. In the following we will focus on the enhancement of the latter key element.

Primaries and multiples are not fully uncorrelated, as they are generated from the same source. This poses a major challenge in the design of an optimal matching filter to minimize the multiple events in the input dataset from their predicted model. Slight differences in their spectra may be exploited with wavelet-based approaches (Pokrovskaia and Wombell, 2004; Ahmed et al., 2007; Neelamani et al., 2008). The present paper follows a similar trail, with a twist towards filter adaptation.

Method

With the aim of simplifying the filter design, we propose to decompose a complicated wide-band problem into a series of more tractable narrow-band issues by means of wavelet frames. Within this approach, the length of the matching filters at each (wavelet) scale can be reduced down to a single sample while still increasing "the matching capability". As this filter is the most computational demanding component of the subtraction element, the reduction in its complexity increases the computational efficiency, despite the use of a redundant representation.

A standard observation model of each 1-D trace of the input dataset can be written as:

$$u[n] = p[n] + s[n] + w[n] \tag{1}$$

where $u[n]$ denotes the recorded signal, $p[n]$ the primary events, $s[n]$ the multiple events and $w[n]$ the background noise. To decompose each trace of the data and the model into the time-scale domain, we have chosen wavelet frames (Daubechies, 1992; Casazza, 2000; Mallat, 2009) that approximate the complex Morlet wavelet transform. The Morlet wavelet can be written as:

$$\psi(t) = \pi^{-1/4} e^{-j\omega_0 t} e^{-t^2/2} \tag{2}$$

where ω_0 is the central frequency of the modulated Gaussian and t stands for the time variable. And the discrete family of functions employed in the frame (*i.e.* a sampling of the continuous wavelet transform) is given by:

$$\psi^{v}_{r,j}[n] = \frac{1}{\sqrt{2^{j+v/V}}}\,\psi\left(\frac{nT - r2^{j}b_0}{2^{j+v/V}}\right) \tag{3}$$

with $r, j \in \mathbb{Z}$, r being the (discretized) time, j the number of scale, b_0 the sampling period at the scale zero, V the number of voices per octave and $v \in [0, V-1]$ the octave. Using the above wavelet family, the time-scale representation of a given trace $u[n]$ is given in closed form by the scalar product formula: $Wu_{j,v}[r] = \langle u, \psi_{r,j,v}\rangle$.

The main problem to solve is the design of an optimum unary (one-sample) filter which minimizes the error between multiple events and their model, leaving primary events unharmed. Once the multiples are estimated and subtracted from the original data in time-scale, the filtered trace is synthesized in the time domain applying an inverse Morlet wavelet transform.

Following a minimum energy approach, the optimum complex value a is the one that, multiplied with the decomposed multiple model $Ws_{j,v}[r]$, minimizes the mean square error $e(a)$ with the decomposed input dataset $Wu_{j,v}[r]$:

$$a_{\mathrm{opt}} = \min_a e(a) = \min_a \sum_r \left| Wu_{j,v}[r] - aWs_{j,v}[r] \right|^2 \tag{4}$$

where sequences $Wu_{j,v}[r]$ and $Ws_{j,v}[r]$ are complex and the $\sum$ symbol denotes a locally weighed sum along N consecutive samples around time index r.

The optimum value in the mean square error sense is the unary Wiener filter for complex signals:

$$a_{\mathrm{opt}} = \frac{\sum_r Wu_{j,v}[r] Ws^*_{j,v}[r]}{\sum_r \left| Ws_{j,v}[r] \right|^2} \tag{5}$$

This algorithm can either be applied locally or globally, depending on the problem constrains.

At this step, note that proceeding with care is important. Instead of performing a direct measurement of the fractional delay, or equivalently of the group delay, we attempt to estimate the phase delay at each scale or frequency component. When the signal-to-noise ratio (SNR) is high, all the meaningfull phase components are sufficiently well estimated. Alternatively, when the SNR decreases, the moderate phase errors made at key scale components could lead to huge errors on the group delay. As a consequence, a careful processing of the a_{opt} at each scale has to be performed, to provide a low dip varying phase based on the meaningful components.

When the group delay difference between the input and the reference sequences is less than the half the signal period at all scales, it is possible to correct this delay with the previously mentioned approach, because the crosscorrelation between the two sequences is always close to the zero delay. But as the group delay difference increases over this limit, the crosscorrelation peak moves away from the zero delay sample and, as a result, the estimation of a_{opt} done in Equation 5 is no longer possible. As a consequence, a redesign of the filter length is necessary.

A simple way to yield a filter robust to higher delays, while keeping the minimum mean square error criterion, consists in the introduction of a delay term into the above unary filter. This is equivalent to finding the optimum value a and delay m that minimize the following equation:

$$e(a,m) = \sum_r \left| Wu_{j,v}[r] - aWs_{j,v}[r-m] \right|^2 , \tag{6}$$

where the optimum a depending on m, as shown above, is:

$$a_{\mathrm{opt}}[m] = \frac{\sum_r Wu_{j,v}[r] Ws^*_{j,v}[r-m]}{\sum_r \left| Ws_{j,v}[r-m] \right|^2} \tag{7}$$

Several criteria can be chosen in the selection of the optimum delay that are well adapted to the nature of the seismic signals. For example, we can keep with minimum mean square error criterion and find the optimum integer delay m, estimating the a_{opt} at each delay first, and then select the one which minimizes Equation 6. Or on the contrary, we can define the optimum delay as the one that maximizes the normalized crosscorrelation between the adapted model and the recorded signal, commonly called coherence (Neidell and Taner, 1971; Taner et al., 1979; Schimmel and Paulssen, 1997), due to the importance of shape over amplitude.

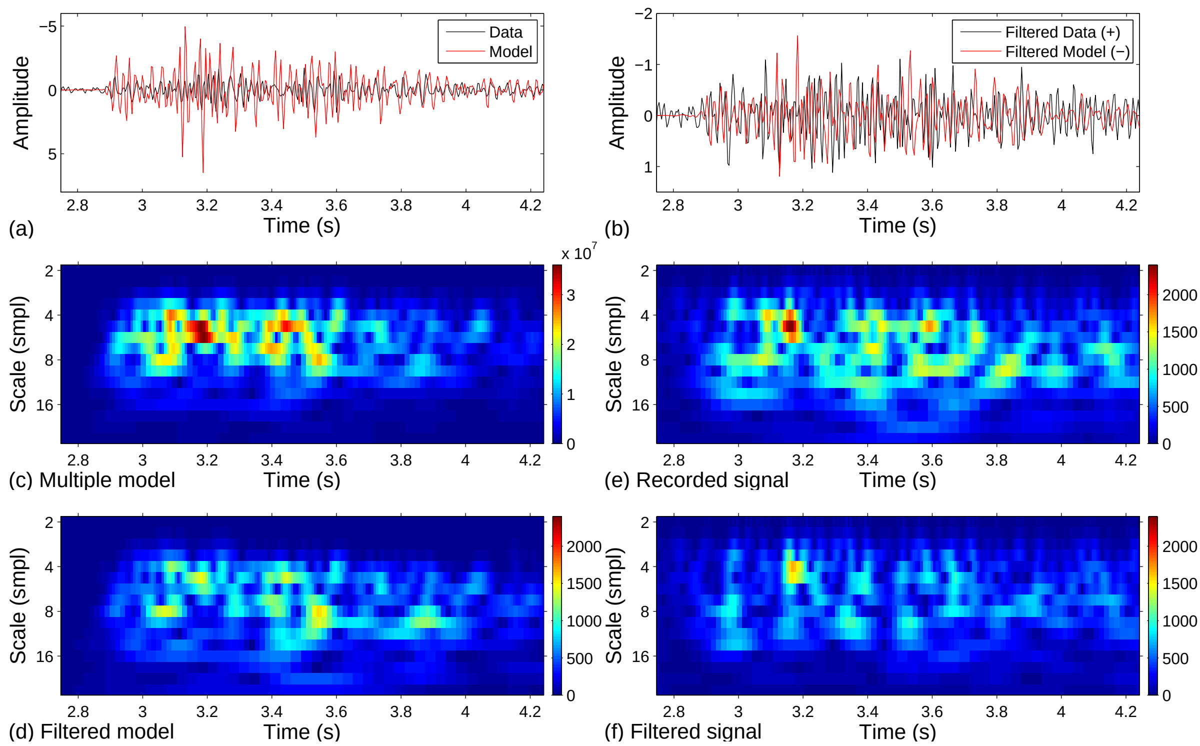

Figure 1 *Subtraction algorithm details. Portion of a trace from the recorded signal, the multiple model, and the filtered multiple model and recorded signal. In time and in time-scale (modulus). The signals in (a) and (b) are attenuated by a factor of 1000 except the multiple model that is* 5×10^7*. The sampling rate is 250 smpl/s.*

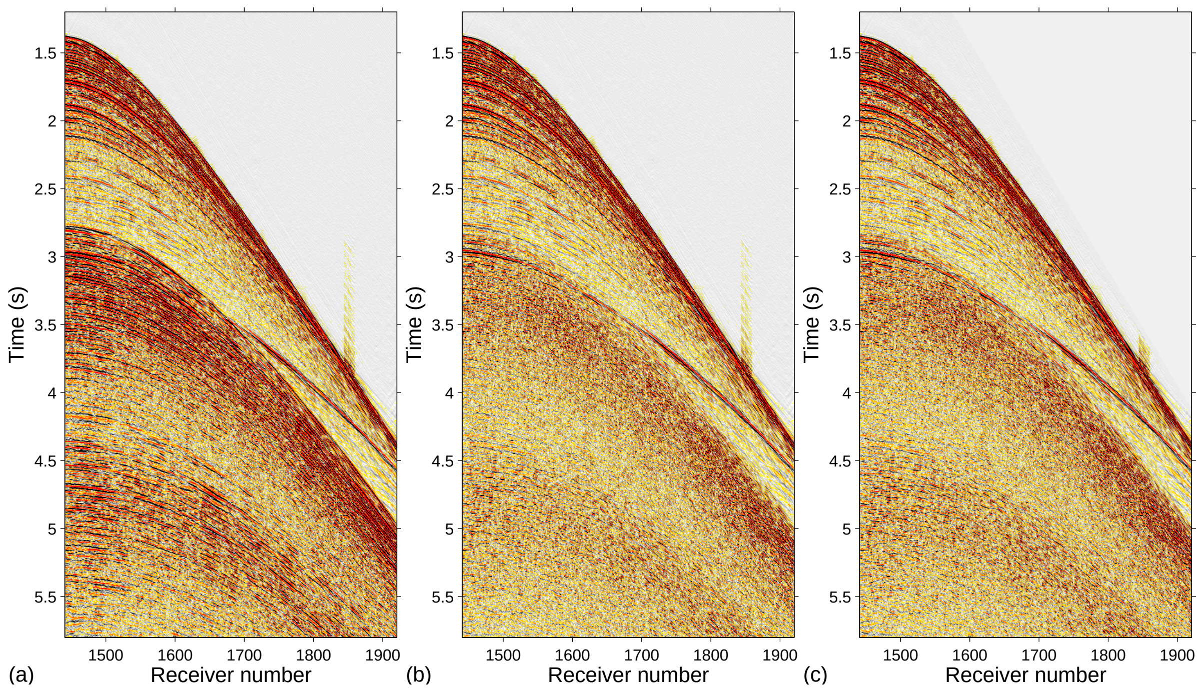

Figure 2 *Subtraction algorithm results. (a) Recorded signal. (b) Results with the 1-D unary complex filters in the time-scale domain. (c) Results with a standard 2-D adaptive filter in time-space domain.*

Results

The example shown in Figures 1 and 2 is taken from a 2-D real marine dataset. In both figures the data is decomposed with a Morlet wavelet with $\omega_0 = 6.3$ rad/smpl, $j \in [1,4]$, $v \in [0,3]$ and $b_0 = 0.5$. The optimum unary filter estimation for each sample uses a rectangular window of 0.636 s. Additionally, to provide robustness against delays higher than half of the signal period, we have chosen the maximum normalized crosscorrelation criterion with a range of ± 12 ms.

Figure 1 provides some intermediate results on a portion of a trace about the first multiple arrival. On the left panel the recorded signal and the multiple model are plotted in time, together with their modulus in the time-scale domain; while on the right panel the filtered model and the filtered recorded signal. As shown in the time-scale domain figures, the unary filter has successfully reduced the differences between the real multiple events and their predicted model to attenuate the multiple events in the recorded signal enough to uncover the main primary events. The main objective in Figure 2(a) is to uncover the primaries that are masked by the strong first multiple events using the 1-D unary filters and standard 2-D adaptive filter in time-space domain. As can be appreciated in Figures 2(b) and 2(c), despite not using the additional information that the neighboring traces provide, our 1-D subtraction approach gives results of the same quality as the standard 2-D adaptive filter in time-space domain. Our 1-D technique achieve slightly better levels of noise, while the 2-D a slightly better multiple attenuation.

The wavelet frame provides an inherent scale-adaptive windowing. Combined with scale-wise unary filter estimation it offers a potentially seamless matching procedure in 1-D. It may be additionally constrained by scale-dependent phase filtering. Further developments along the space dimension are being pursued to reinforce the lateral coherence of the method.

Acknowledgments

The authors thank Statoil for allowing us to show the Norwegian Sea results. They also acknowledge CGGVeritas for the authorization to present this work, and are especially grateful to Irène Huard, Sylvain Leroy and Antonio Pica for their help and comments.

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