

# RECENT DEVELOPMENTS FOR MONO- OR MULTI-RATE PARALLEL REAL-TIME CO-SIMULATION

## EXTRAPOLATION AND SCHEDULING FOR MULTICORE ARCHITECTURES

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# OUTLINE

- Background
  - **Co-simulation: context & challenges**
  - Real-time Co-Simulation: an open problem
- Improving parallelism with the RCosim approach: Refined Co-simulation
- Ensuring co-simulation accuracy with CHOPtrey extrapolation approach
- Mapping real-time constraints for HiL
- Future work

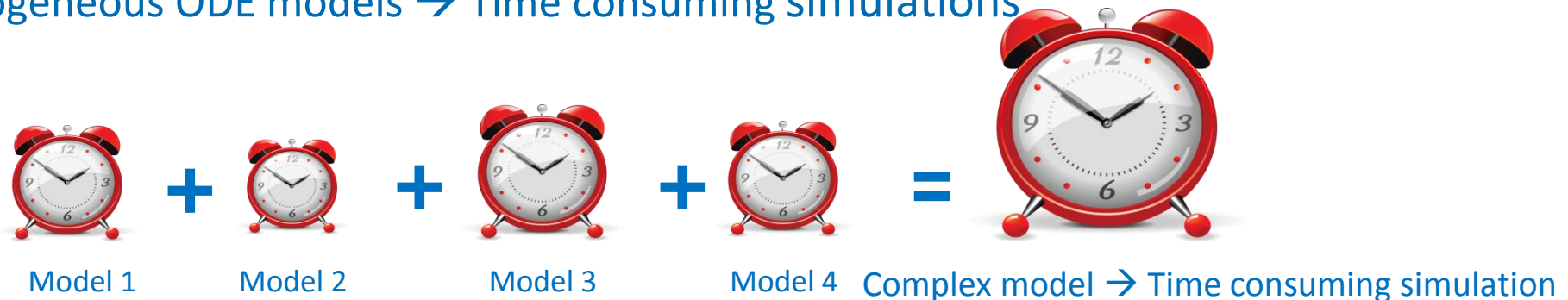
## BACKGROUND

- Co-simulation: Alternative to monolithic simulation → Simulation of a complex system using several coupled subsystems
  - A subsystem is modeled using the most appropriate tool instead of using a single modeling software
  - Subsystems are modeled and run in a segregated manner → The equations of each model are integrated using a solver separately
  - During the execution models exchange data → A synchronization mechanism is used between the models, in such a way that models update their inputs and outputs according to assigned communication steps
  - Easy upgrade, reuse, and exchange of models



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  - Easy upgrade, reuse, and exchange of models
  - Heterogeneous ODE models → Time consuming simulations



## BACKGROUND (CONT'D)

- A multi-core co-simulation kernel: Why?
  - System-level simulation leads to agglomerate models which are classically disconnected, increasing the CPU demand at simulation time
  - Simulation time becomes more and more a metric for model complexity
  - Most 0D/1D simulation tools have mono-core kernel while mono-core computers are endangered

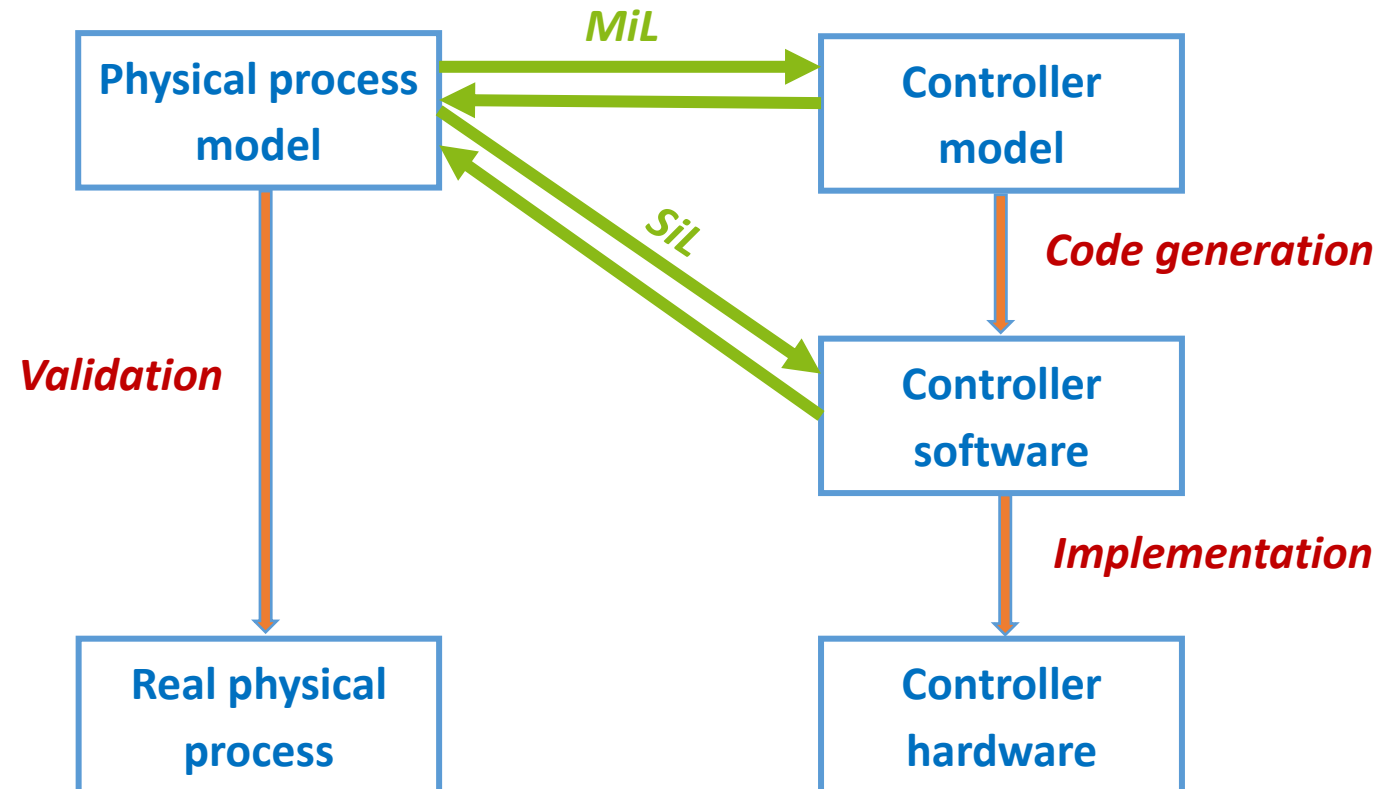
How long will this CPU power remain unused ?



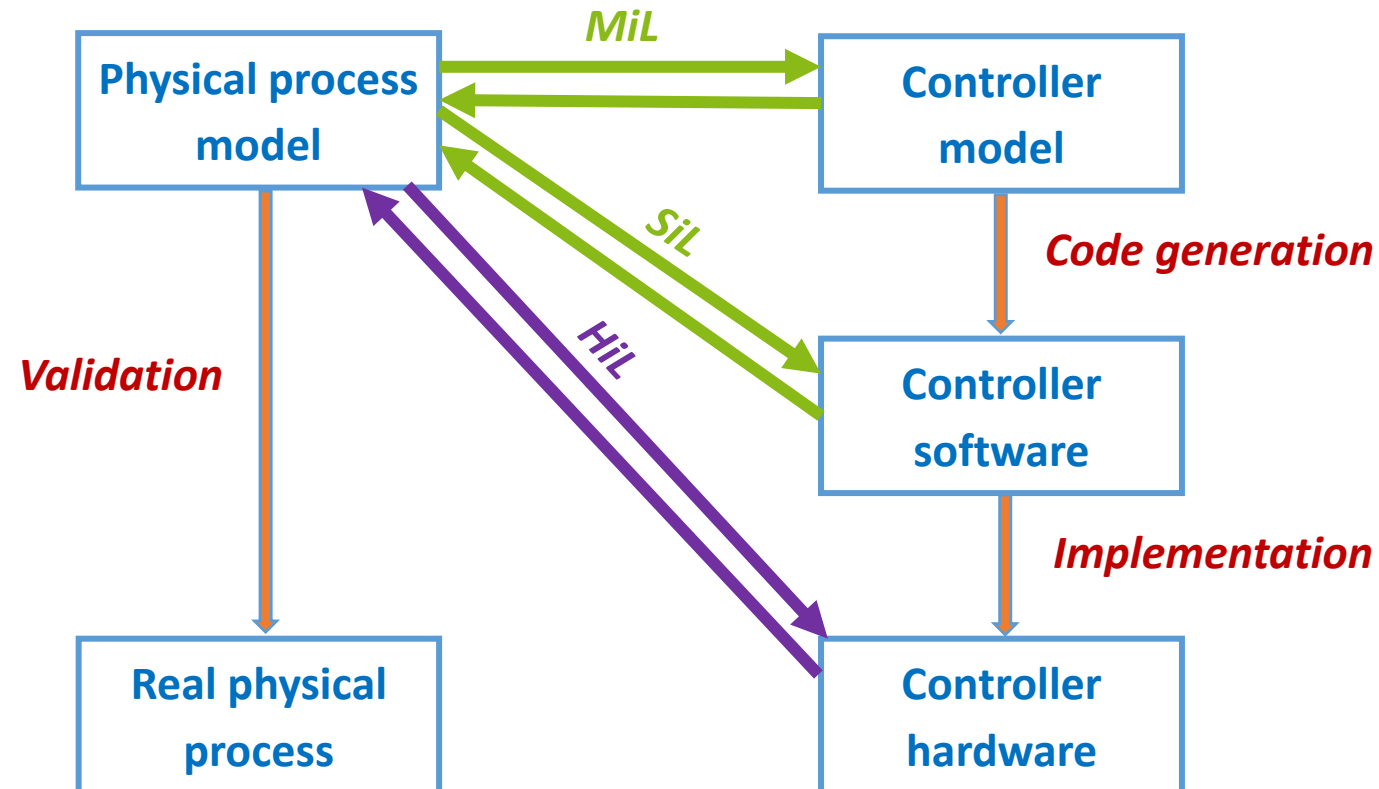
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# REAL-TIME SIMULATION NEEDS FOR CPS VALIDATION



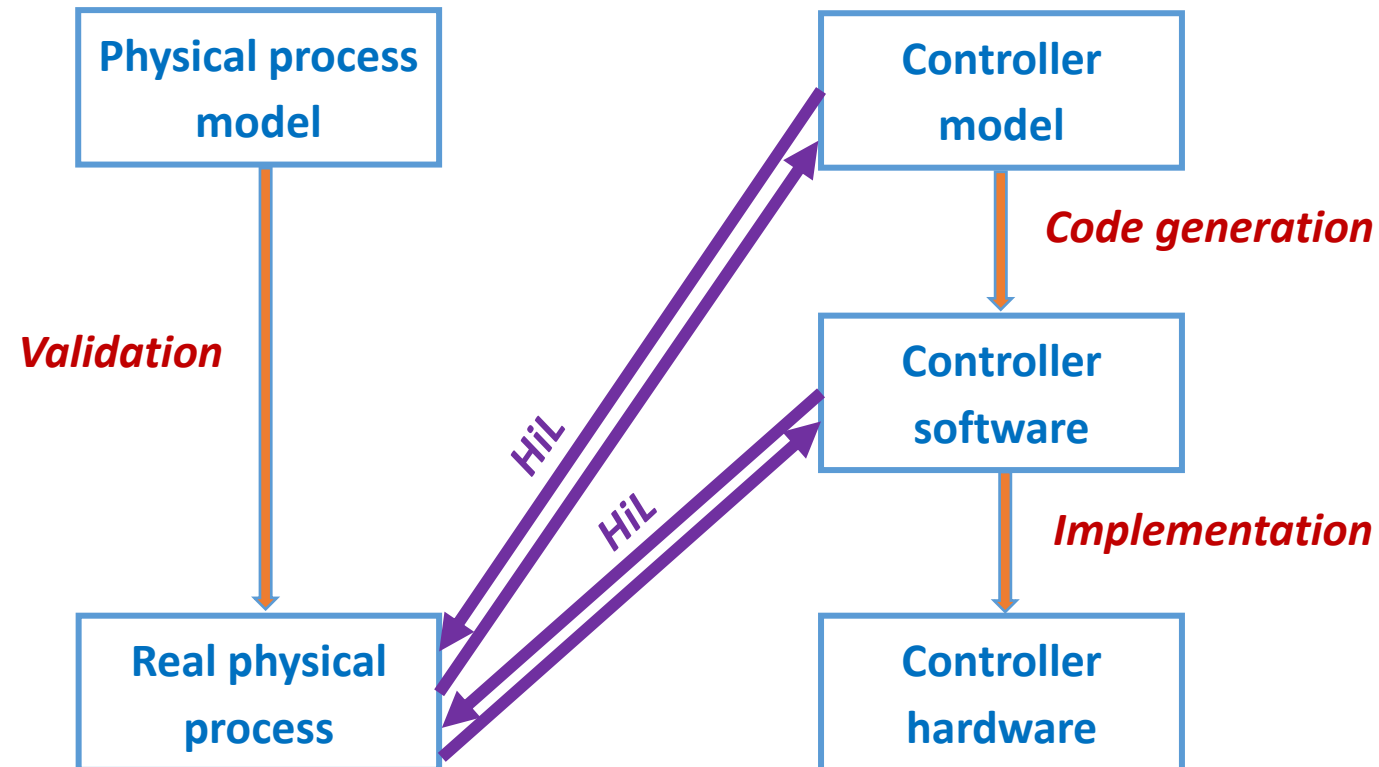
# REAL-TIME SIMULATION NEEDS FOR CPS VALIDATION



Hardware-in-the-Loop → real-time constraints

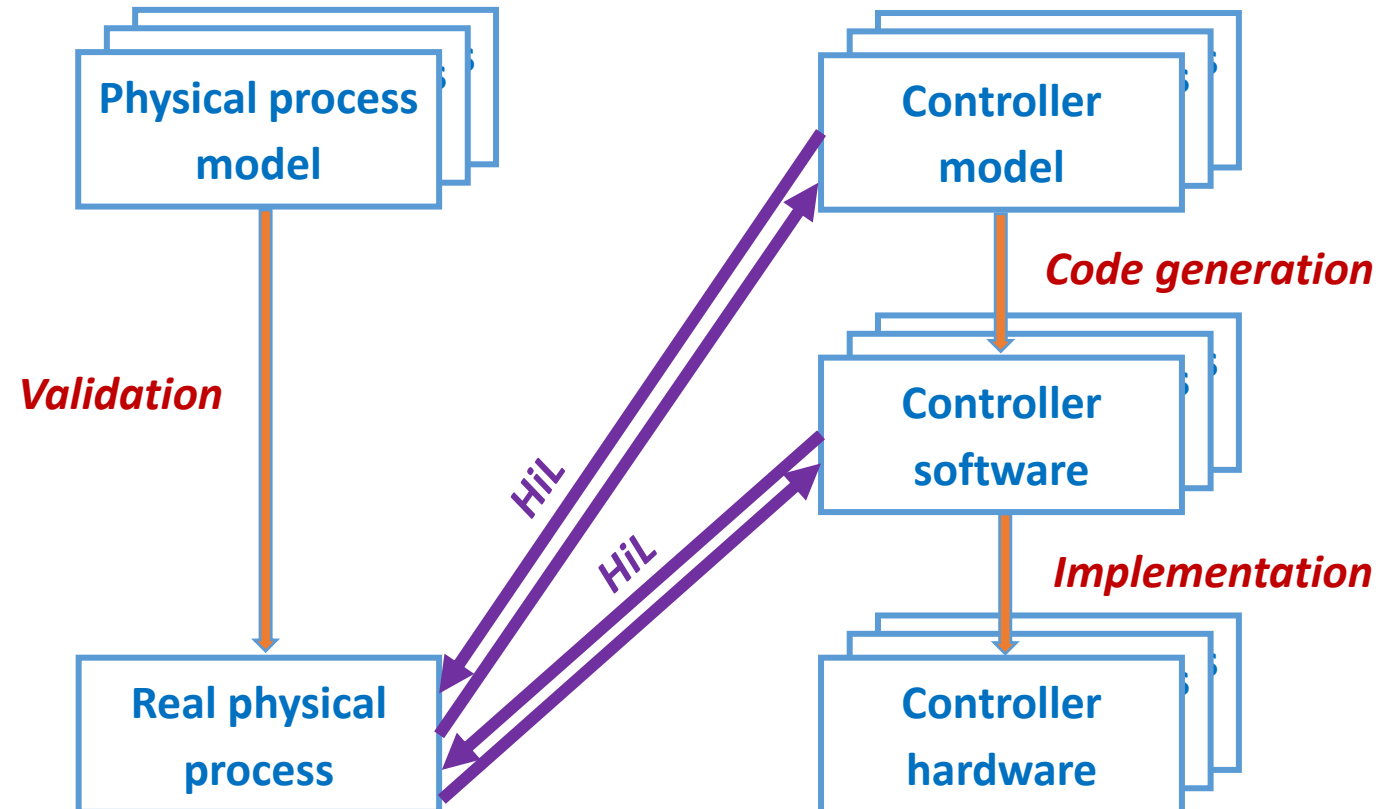


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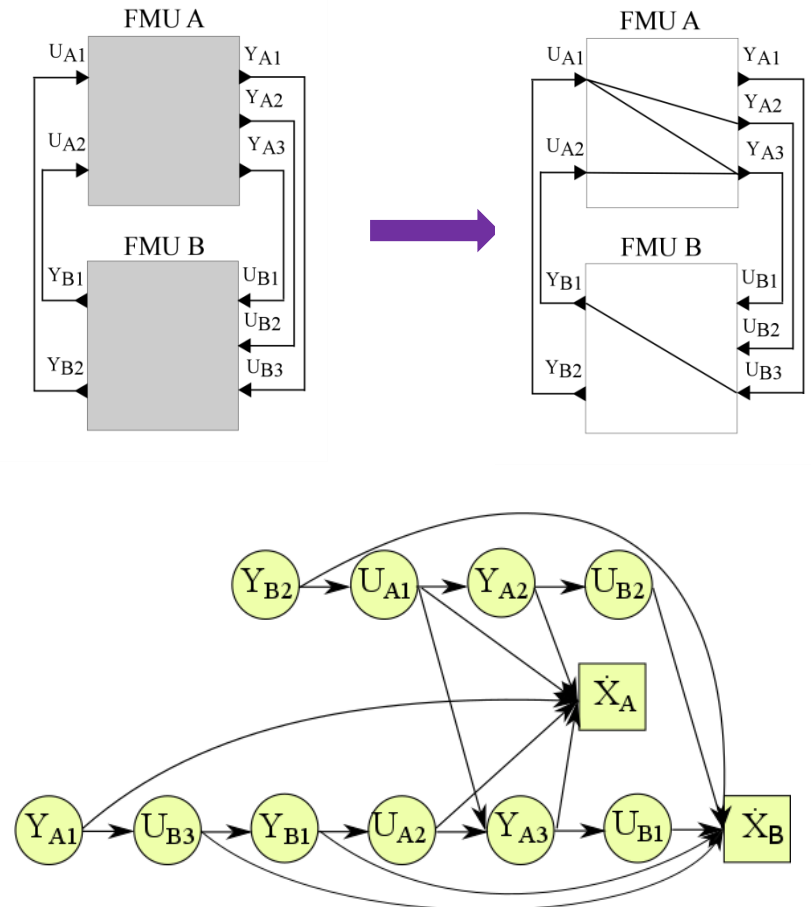
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- Mapping real-time constraints for HiL
- Summary and outlook

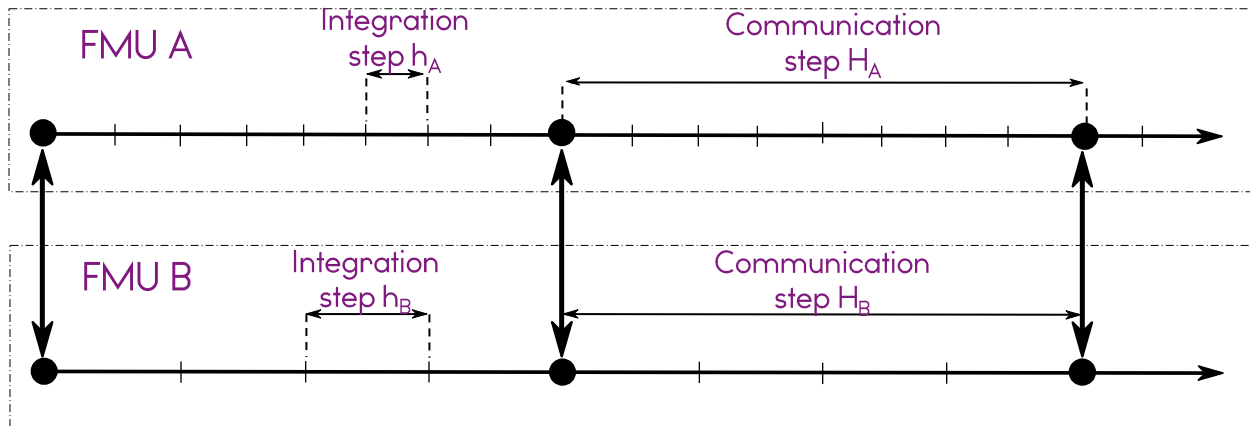
# RCOSIM: REFINED CO-SIMULATION DATAFLOW GRAPH OF FMUS

- Inter FMU dependencies specified by the user
- Identify locally if Y is dependent on U or not
  - FMI gives relationships between each Y and U
  - With FMI each I/O is computed with a different FMU functions
- Build refined dependency graph
  - Vertices: operations, a set of FMU functions
    - updateOut, updateIn, and updateState
  - Directed edges: precedencies between operations
  - Ordinary Differential Equations (ODEs)
    - No algebraic loops
    - Directed Acyclic Graph (DAG)
- Apply a multi-core scheduling heuristic on the dataflow graph



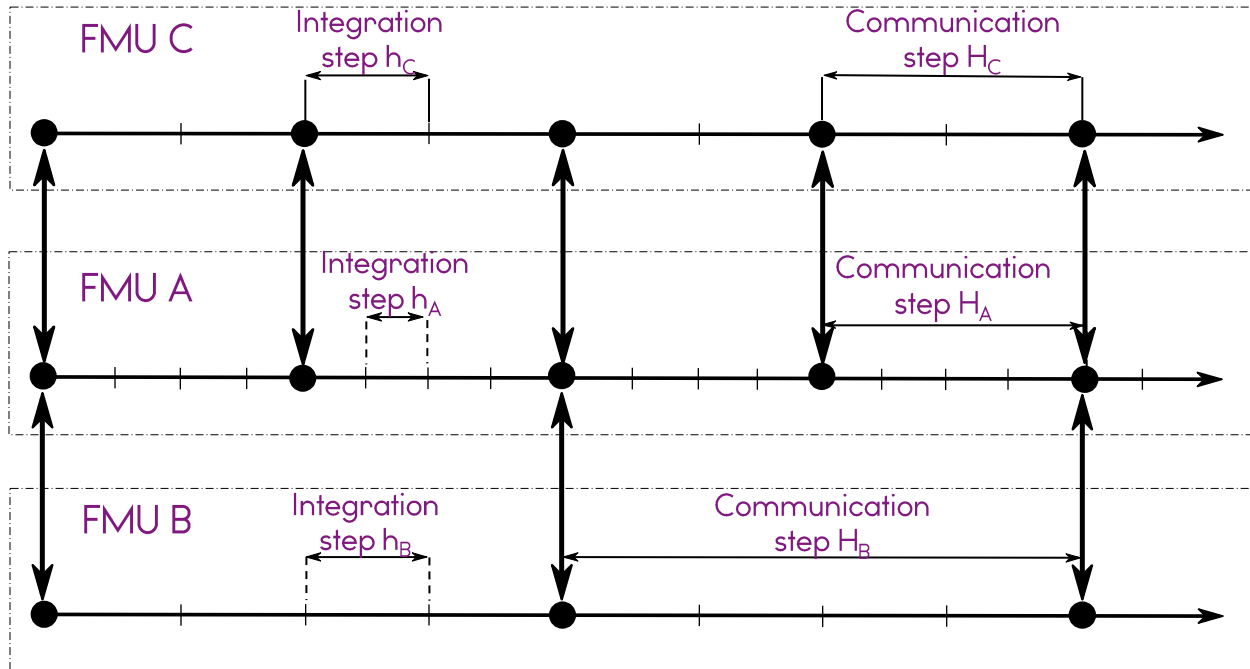
# MUO-RCOSIM

## EXTEND RCOSIM TO HANDLE MULTI-RATE CO-SIMULATION



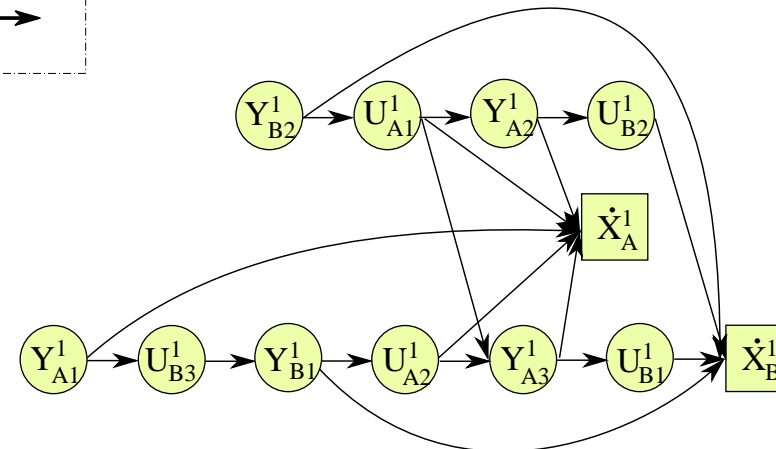
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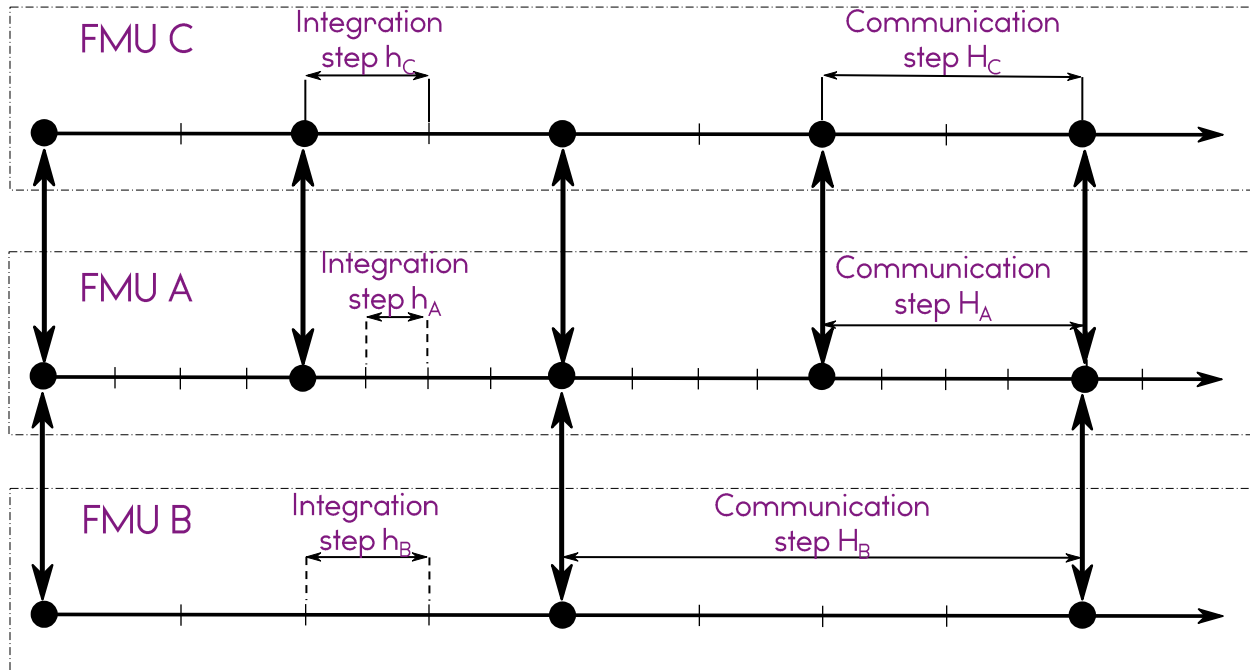
### Multi-rate co-simulation

- Update the I/O of each FMU according to its communication step
- Need for a repeatable pattern of the multi-rate graph execution
- Repeat each operation  $r_i = HS/H(o_i)$  times,  $HS = \text{lcm}(H(o_1), H(o_2), \dots, H(o_n))$
- E.g:  $H_B = 2 \times H_A$



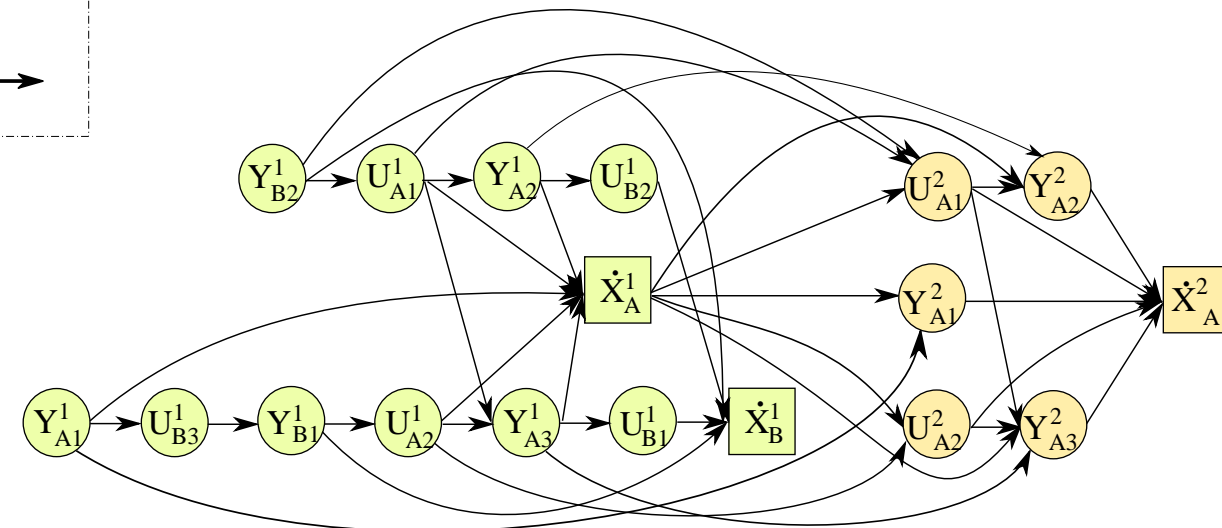
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Type of dependency	Slow to Fast	Fast to Slow
Communication steps	$H(o_i) > H(o_j)$	$H(o_i) < H(o_j)$
Edge creation rule	$o_i^s \rightarrow o_j^u,$ $s \in \{1, 2, \dots, r_i\}$ $u = \left\lceil s \times \frac{H(o_i)}{H(o_j)} \right\rceil$	$o_i^s \rightarrow o_j^u,$ $u \in \{1, 2, \dots, r_j\}$ $s = \left\lceil u \times \frac{H(o_j)}{H(o_i)} \right\rceil$

# MUO-RCOSIM

## MULTI-RATE GRAPH TRANSFORMATION (CONT'D)

### Multi-Rate Graph Transformation Algorithm

- 1) Compute the hyper-step  $HS = \text{lcm}(H(o_1), H(o_2), \dots, H(o_n))$
- 2) For each operation  $o_i$  in the graph
  - Compute the repetition factor  $r_i = HS / H(o_i)$
- 3) Repeat each operation  $o_i$ ,  $r_i$  times
- 4) Add edges between successive occurrences of each operation
- 5) For each edge  $(o_i, o_j)$ 
  - If  $H(o_i) > H(o_j)$  (slow to fast dependency)
 
$$\text{Add edges } (o_i^s, o_j^u), s \in \{1, 2, \dots, r_i\}, u = \left\lfloor s \times \frac{H(o_i)}{H(o_j)} \right\rfloor$$
  - If  $H(o_i) < H(o_j)$  (fast to slow dependency)
 
$$\text{Add edges } (o_i^s, o_j^u), u \in \{1, 2, \dots, r_j\}, s = \left\lfloor u \times \frac{H(o_j)}{H(o_i)} \right\rfloor$$
  - If  $H(o_i) = H(o_j)$ 

$$\text{Add edges } (o_i^s, o_j^u) \text{ between corresponding occurrences}$$
- 6) For each FMU
  - Add edges between the occurrence  $s$  of the state operation and all the input and output operations of the next occurrence  $s+1$
- 7) Stop when all operations and edges have been visited



## MUO-RCOSIM MULTI-CORE SCHEDULING

- Off-line heuristic approach: Similar to SynDEx (INRIA) [Grandpierre et al., 1999]
- N operations, each one:
  - Computation time
  - Earliest and latest start and end dates → Takes into account the synchronization cost
- Objective: Minimize the makespan (multiprocessor critical path) of the graph
- Cost function: Schedule pressure is the difference between:
  - Flexibility: Freedom degree of an operation: time interval inside which  $o_i$  may be executed without increasing the makespan
  - Penalty: Critical path increase by setting an operation on a processor accounting for synchronization cost

# MUO-RCOSIM

## MULTI-CORE SCHEDULING (CONT'D)

### Multi-core scheduling heuristic

- 1) For each operation  $o_i$ 
  - Compute  $S_i$  (resp.  $E_i$ ) the earliest start (resp. end ) time, and  $S'_i$  (resp.  $E'_i$ ) the latest start (resp. end ) time
  - Compute the flexibility  $F_i = CP - E_i - E'_i$
- 2) Set  $\Omega$  the set of operations without predecessors
- 3) Repeat
  - For each pair (operation  $o_i$  in  $\Omega$ , core  $p_j$ )
    - Compute the increase (cost) of scheduling  $o_i$  on  $p_j$
    - Select for  $o_i$ , the core  $p_j$  which minimizes the cost of scheduling  $o_i$
  - Find the operation  $o_i$  with the maximal cost on its selected core
  - Allocate  $o_i$  to its selected core
  - Remove  $o_i$  from  $\Omega$
  - Add to  $\Omega$  every operation whose predecessors have been scheduled
  - Stop when all the operations have been scheduled

# TESTS

## ● Case study

- Spark Ignition RENAULT F4RT engine
- 6 FMUs, more than 100 operations
- Around 300 operations after applying the multi-rate transformation algorithm

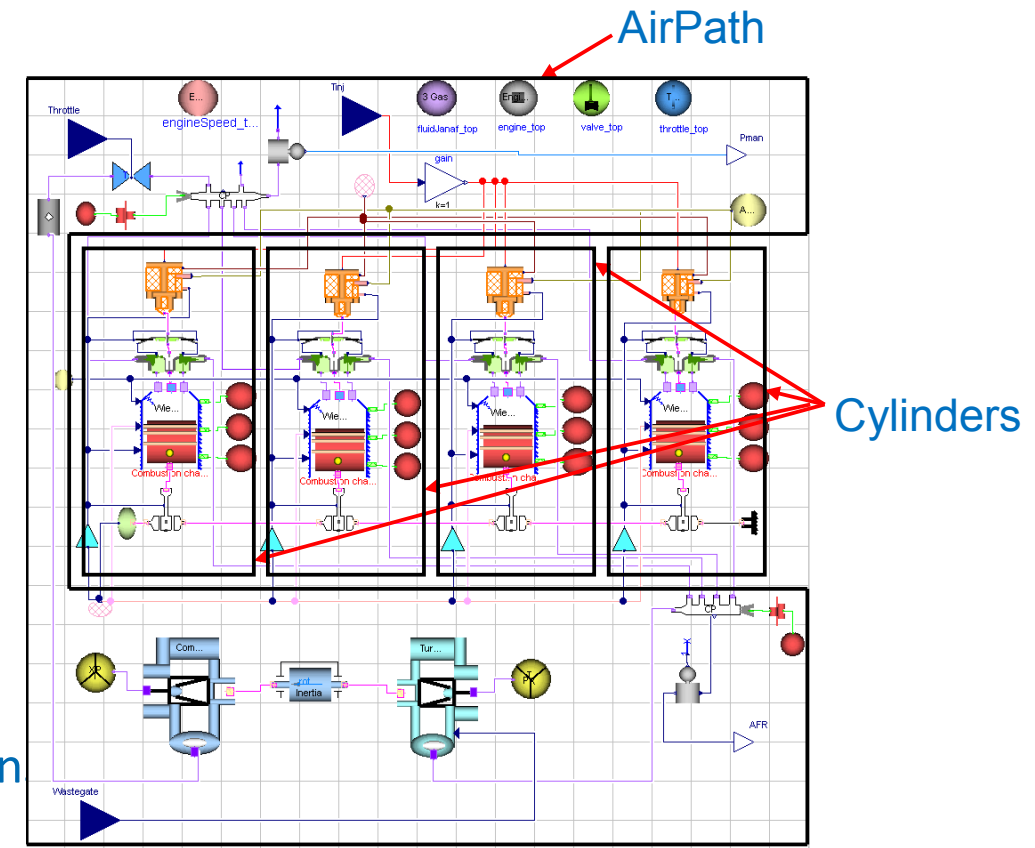
- Communication steps

- Airpath/control: 100  $\mu$ s
- Cylinders: 20  $\mu$ s

- Integration step = communication step for all FMUs

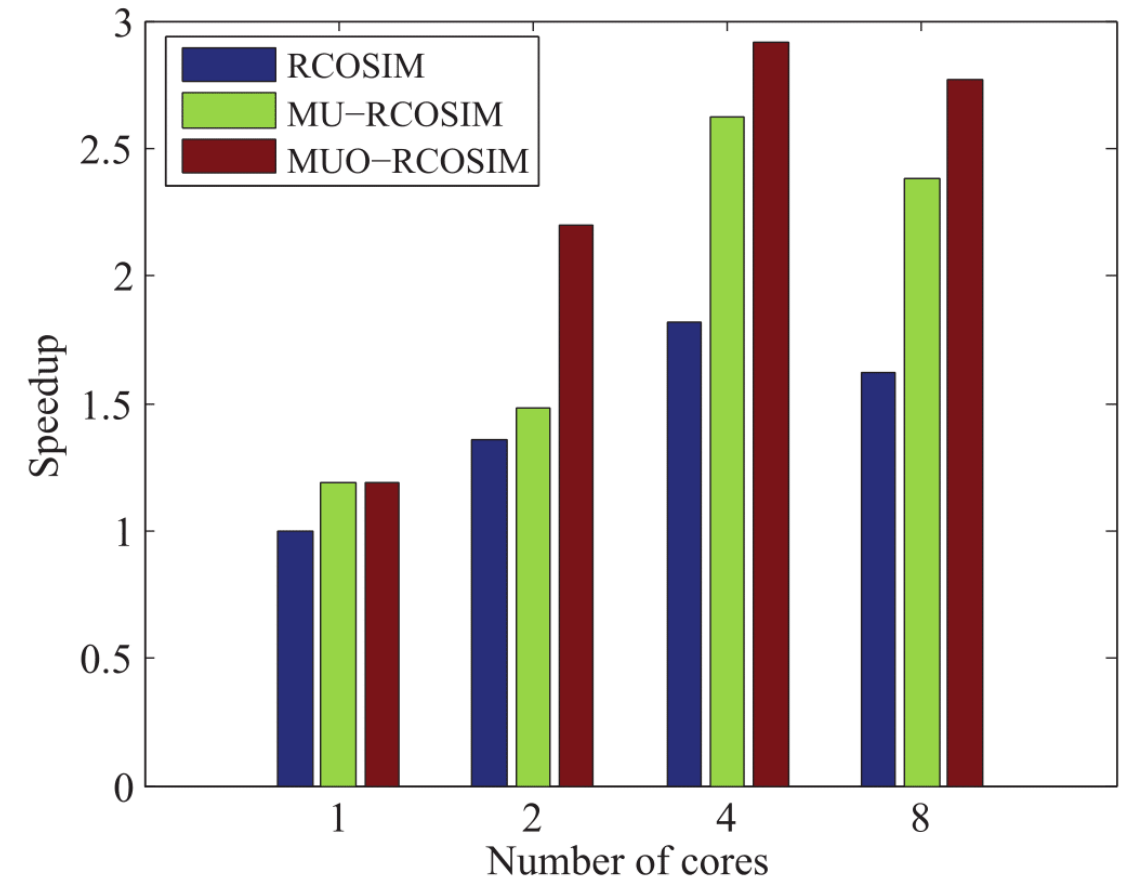
## ● 3 approaches are compared

- RCosim: Mono-rate, restricted allocations of the operation
- MU-RCosim: **M**ulti-rate, restricted allocation of the operations
- MUO-RCosim: **M**ulti-rate, uses the acyclic **O**rientation heuristic to handle mutual exclusion constraints



## TESTS SPEED-UP

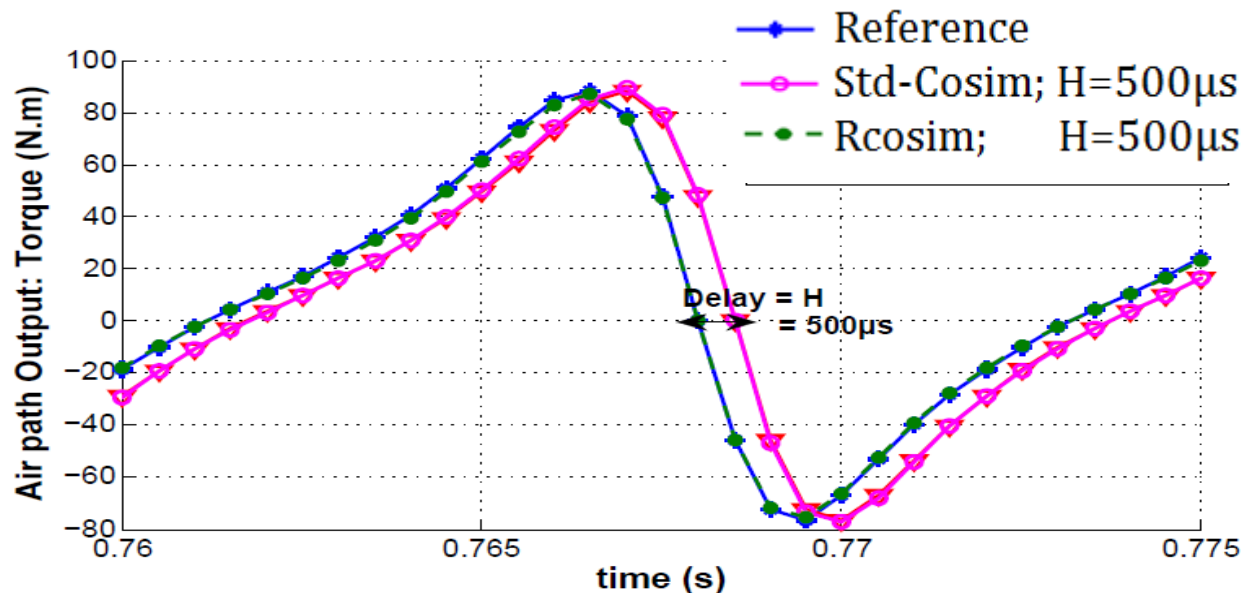
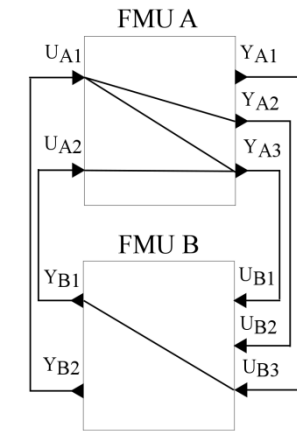
- Speed-up =  $\frac{\text{Sequential execution time}}{\text{Parallel execution time}}$ 
  - Best speed-up close to 2.9 reached with 5 cores (compared to mono-core schedule)
  - MUO-RCosim > MU-RCosim > RCosim
  - Thanks to the mutual exclusion heuristic, an efficient execution order for mutual exclusive operations is defined
  - This order tends to allow the multi-core scheduling heuristic to better adapt the potential parallelism to the execution platform



# RCOSIM APPROACH

## ACCURACY: ELIMINATION OF DELAYS

- Torque is a direct feedthrough output: e.g.  $Y_{A3}$
- Expected delays with Standard Co-simulation (Std-Cosim) due to arbitrary order execution decision between models
- No delays with RCosim
  - The execution order is compliant with initial model



Test with variable step solver: LSODAR

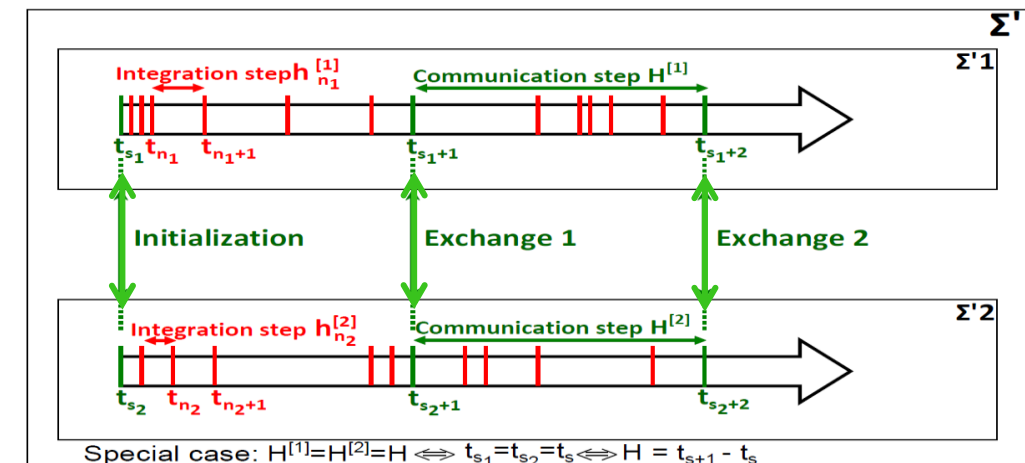
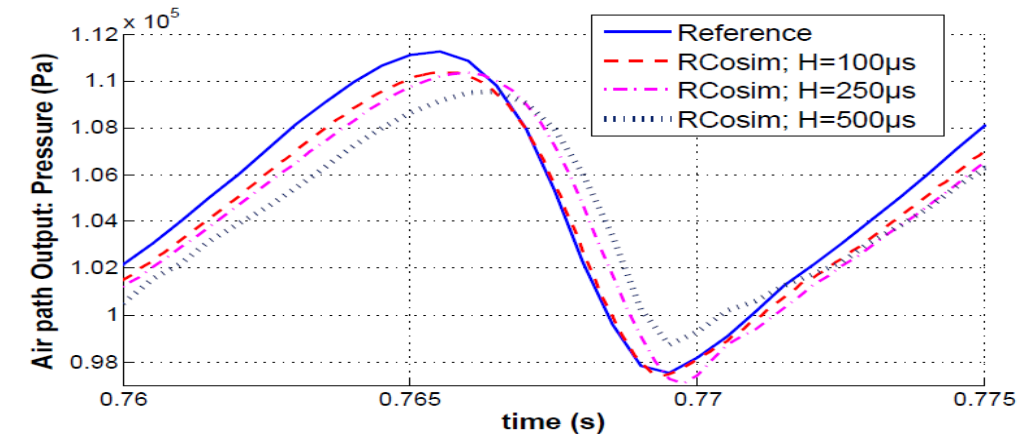
Simulation method	Std-Cosim	RCosim
Er(%) with H=100µs	2.95	0.68
Er(%) with H=250µs	9.12	1.1
Er(%) with H=500µs	<b>19.83</b>	<b>1.37</b>

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# CONTEXT-BASED EXTRAPOLATION IMPROVE AGAIN THE SIMULATION ACCURACY

- Limitation: with RCosim, errors are reduced but still exist
- Reason: Input data is held constant during the communication step
- Dilemma: ↗ ↗ communication step
  - ↗ ↗ Integration error
  - ↗ ↗ Speed-up
- Idea: Extrapolate input signals to
  - Enlarge intervals
  - Reduce simulation errors



## RELATED WORK

### ● Difficulties

- Related work on extrapolations treated the continuous case
  - Successful for non-stiff systems / Encountered problems with stiff systems
- Complex systems with hybrid behavior is even more difficult to predict
  - Nonlinearities, discontinuities,...
- ➔ Hard to predict the future behavior (from past observations)
  - Polynomial prediction fails due to the discontinuities
  - No universal prediction scheme, efficient with every signal

### ● Challenges: fast, causal and reliable prediction

- Predictor computing cost  $\ll$  extra model computations with small communication steps
- Accurate predictions for any signal (blocky/smooth; slow/steep onsets)

### ● Idea: Borrow the context-based approach from lossless image encoders



# CHOPRED: COMPUTATIONALLY HASTY ONLINE PREDICTION

## CHOPOLY: CAUSAL HOPPING OBLIVIOUS POLYNOMIALS

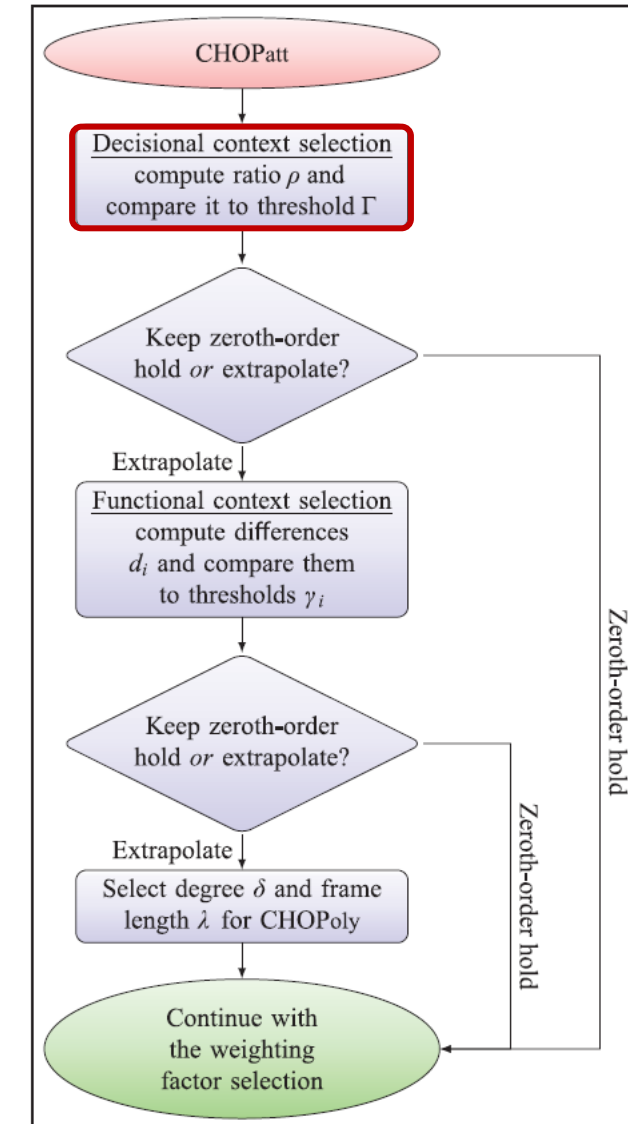
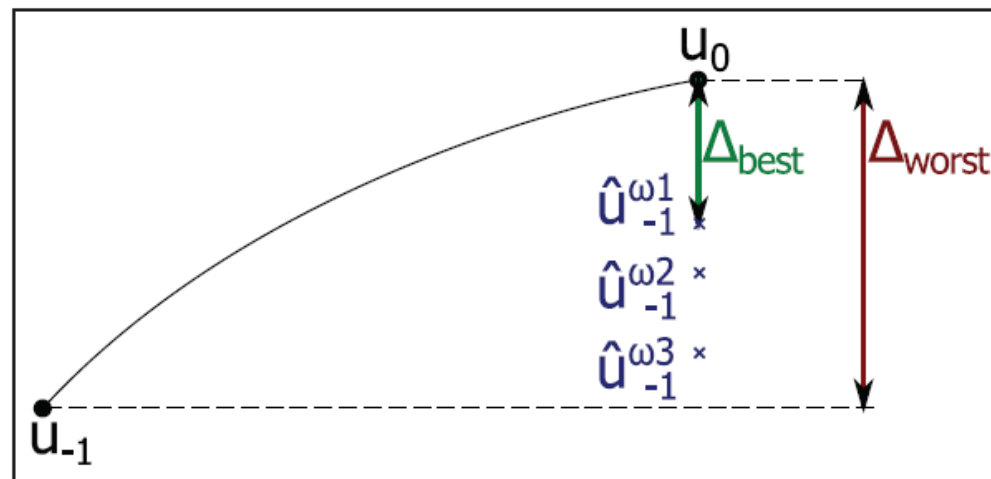
- $P_{\delta,\lambda,\omega}$  : least squares polynomial predictor
  - $\delta$ : prediction degree;
  - $\lambda$ : prediction frame length
  - $\omega$ : weighting factor
- $u$ : input signal;  $\tau$ : relative time for prediction
- Weighted moment:  $\bar{m}_{d,\lambda,\omega} = \sum_{l=0}^{\lambda-1} (\lambda - l)^{\omega} l^d u_{-l}$
- Weighted sum of integer powers:  $\bar{z}_{d,\lambda,\omega} = \sum_{l=0}^{\lambda-1} (\lambda - l)^{\omega} l^d$
- General formula for extrapolation:
  - Use of LUT → fast computation

$$u(\tau) = \begin{bmatrix} 1 & \tau & \dots & \tau^{\delta} \end{bmatrix} \begin{bmatrix} \bar{z}_{0,\lambda,\omega} & -\bar{z}_{1,\lambda,\omega} & \dots & (-1)^{\delta} \bar{z}_{\delta,\lambda,\omega} \\ -\bar{z}_{1,\lambda,\omega} & & & \vdots \\ \vdots & & & \vdots \\ (-1)^{\delta} \bar{z}_{\delta,\lambda,\omega} & \dots & \dots & \bar{z}_{2\delta,\lambda,\omega} \end{bmatrix}^{-1} \begin{bmatrix} \bar{m}_{0,\lambda,\omega} \\ -\bar{m}_{1,\lambda,\omega} \\ \vdots \\ (-1)^{\delta} \bar{m}_{\delta,\lambda,\omega} \end{bmatrix}$$

# CHOPatt: CONTEXTUAL AND HIERARCHICAL ONTOLOGY OF PATTERNS

## META- OR DECISIONAL CONTEXT SELECTION

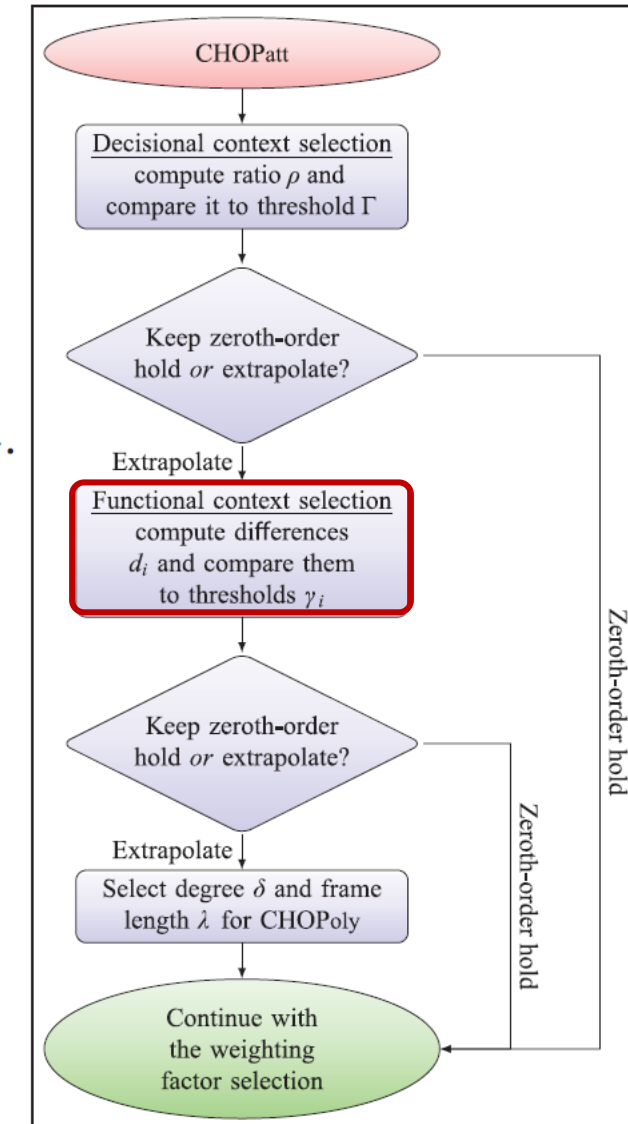
- Worst case scenario without extrapolation:  $\Delta_{\text{worst}} = |u_0 - u_{-1}|$
- Best prediction pattern:  $\Delta_{\text{best}} = \min_{\omega \in \Omega} |u_0 - \hat{u}_{-1}^{\omega}|$  ;  $\Omega = \{0, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2\}$
- Ratio:  $\rho = \frac{\Delta_{\text{best}}}{\Delta_{\text{worst}}}$
- Threshold:  $0.7 \leq \Gamma < 1$  e.g.  $\Gamma = 90\%$
- If  $\rho > \Gamma$  then sharp and fast variation  $\rightarrow$  Select the decisional context: cliff context



# CHOPatt: CONTEXTUAL AND HIERARCHICAL ONTOLOGY OF PATTERNS

## FUNCTIONAL CONTEXT SELECTION

- Differences (variations):  $d_0 = u_0 - u_{-1}$  and  $d_{-1} = u_{-1} - u_{-2}$
- Thresholds:  $\gamma_0 = \gamma_{-1} = \frac{1}{2} \max_{i \in [1-\Lambda, \dots, -3]} (|u_i - u_{i+1}|)$
- Conditions:  $O$  if  $|d_i| = 0$ ;  $C_i$  if  $0 < |d_i| \leq \gamma_i$ ;  $\overline{C}_i$  if  $|d_i| > \gamma_i$ .

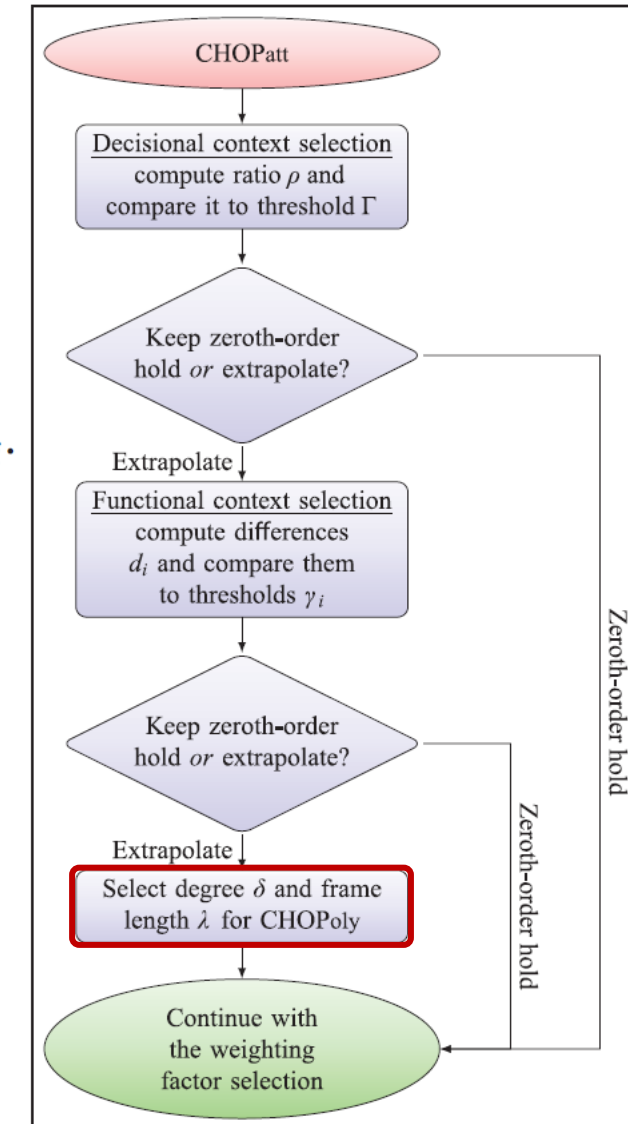
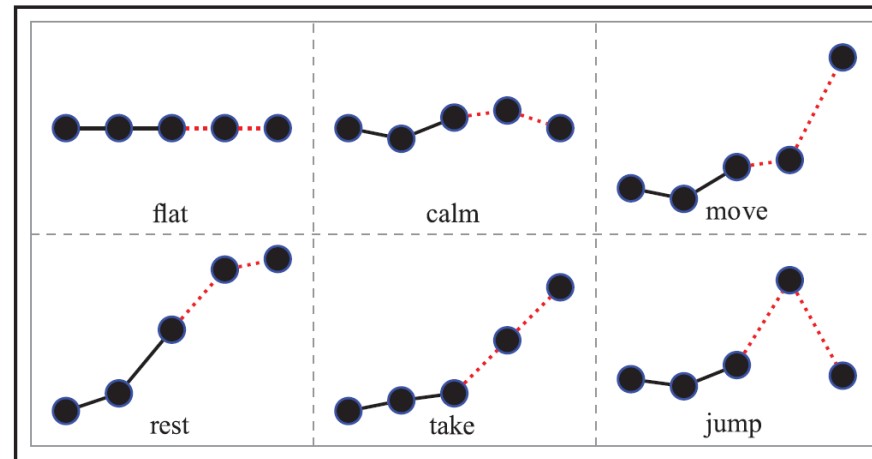


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n(ame)	#	$ d_{-1} $	$ d_0 $	$d_{-1} \cdot d_0$	$(\delta, \lambda, \omega)$
f(lat)	0	$O$	$O$	$O$	$(0, 1, .)$
c(alm)	1	$C_1$	$C_2$	any	$(2, 5, .)$
m(ove)	2	$C_1$	$\bar{C}_2$	any	$(0, 1, .)$
r(est)	3	$\bar{C}_1$	$C_2$	any	$(0, 2, .)$
t(ake)	4	$\bar{C}_1$	$\bar{C}_2$	$> 0$	$(1, 3, .)$
j(ump)	5	$\bar{C}_1$	$\bar{C}_2$	$< 0$	$(0, 1, .)$



# SIMULATION RESULTS WITH CHOPtrey

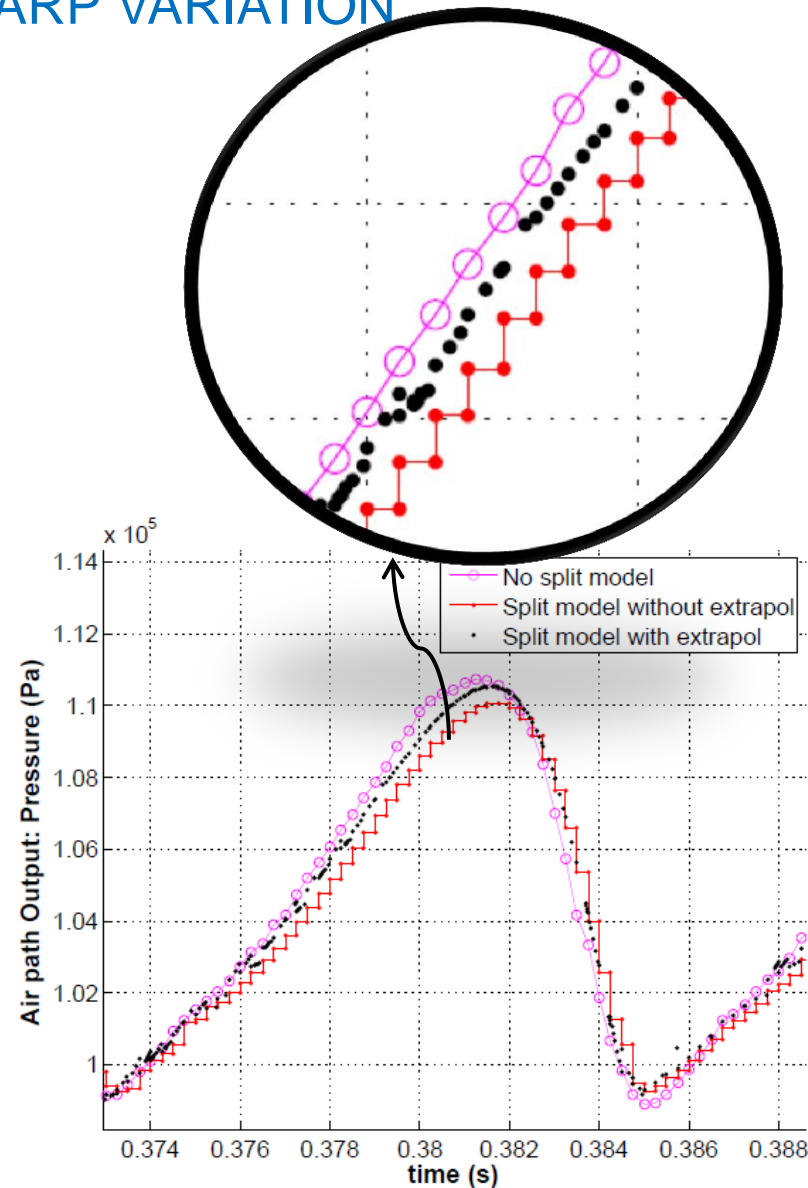
## AUTOMATIC DETECTION OF SHARP VARIATION

- Same case study
  - 118 states/312 events
  - Solver: LSODAR
  - Communication step: 200μs
- Conventional 1<sup>st</sup> & 2<sup>nd</sup> order extrapolation
  - Fails on the engine model
  - Major causes:
    - Discontinuities
    - Sharp variations

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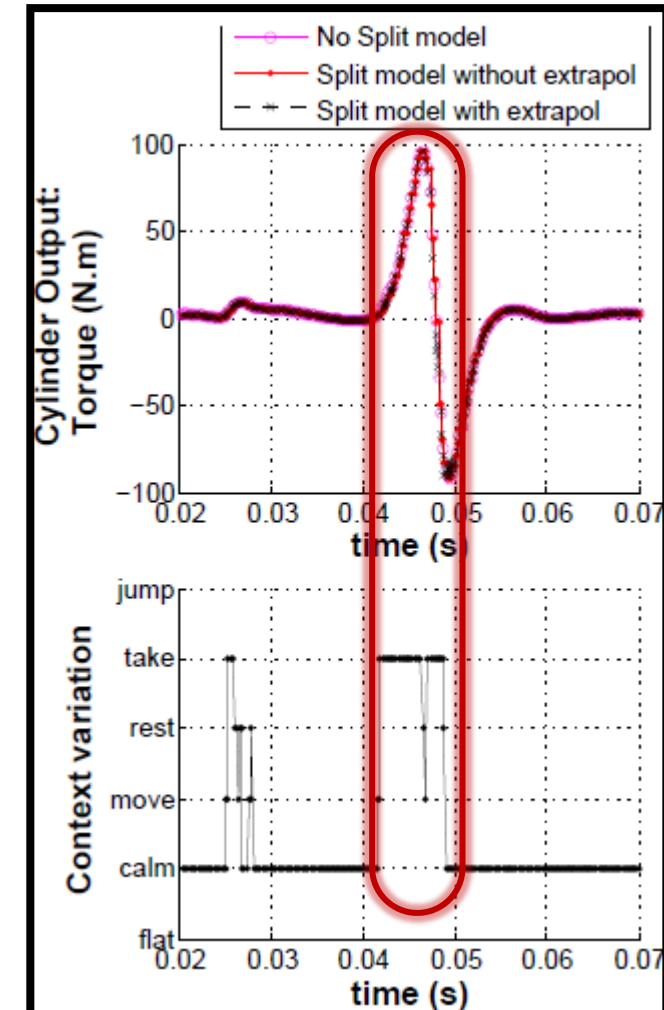
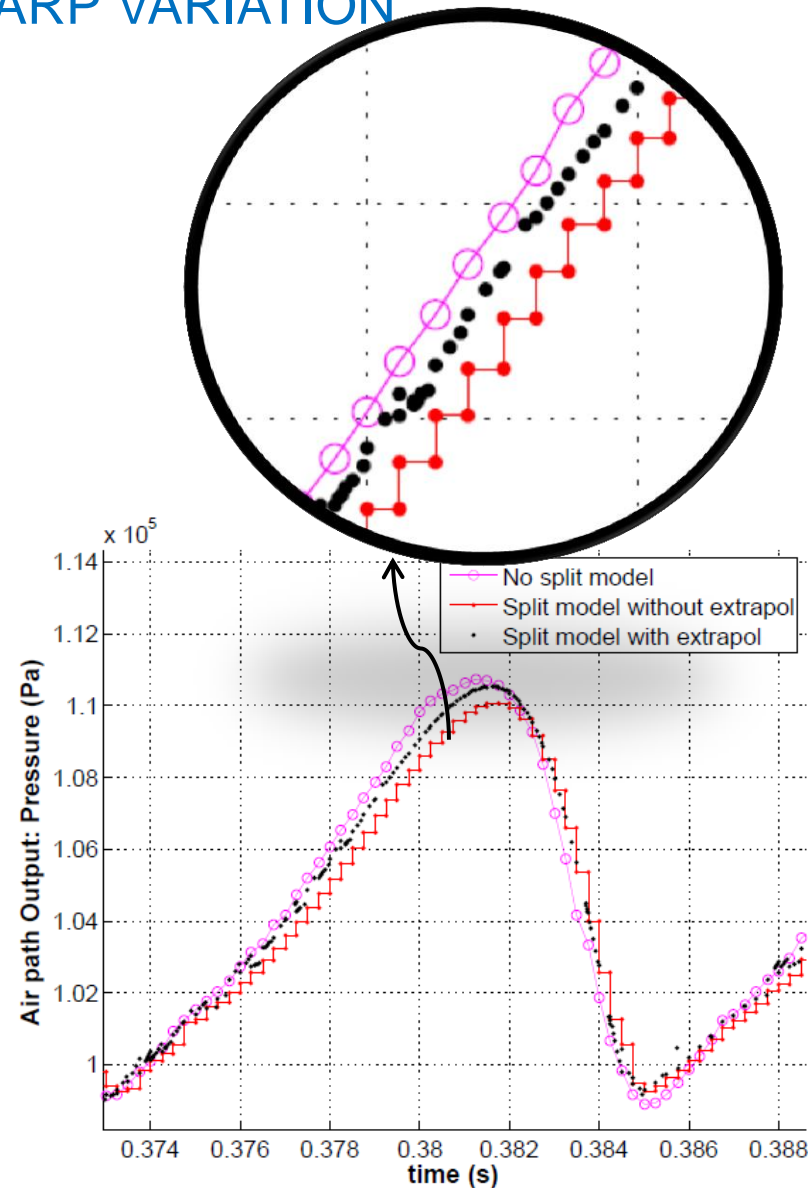
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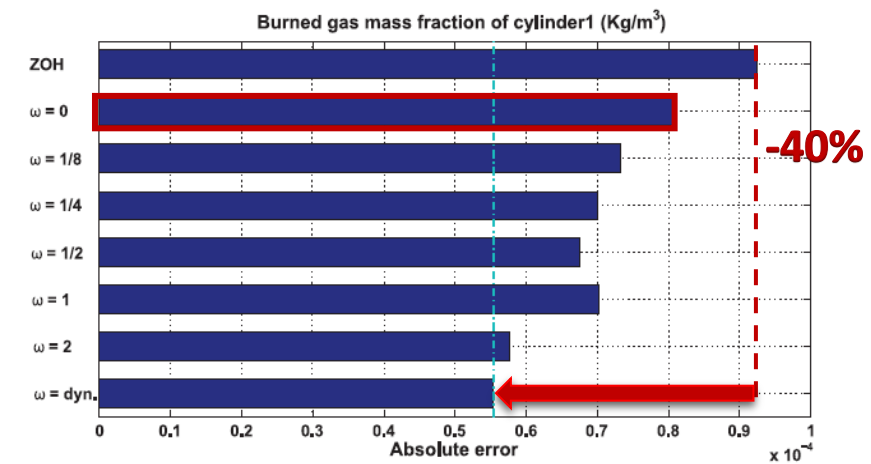
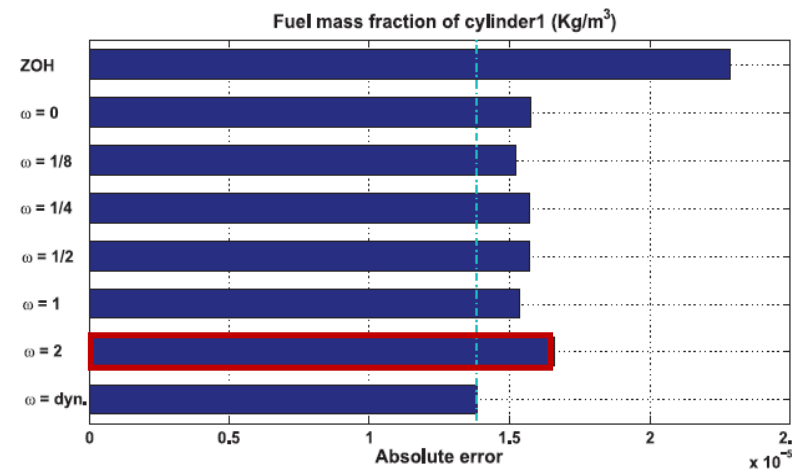
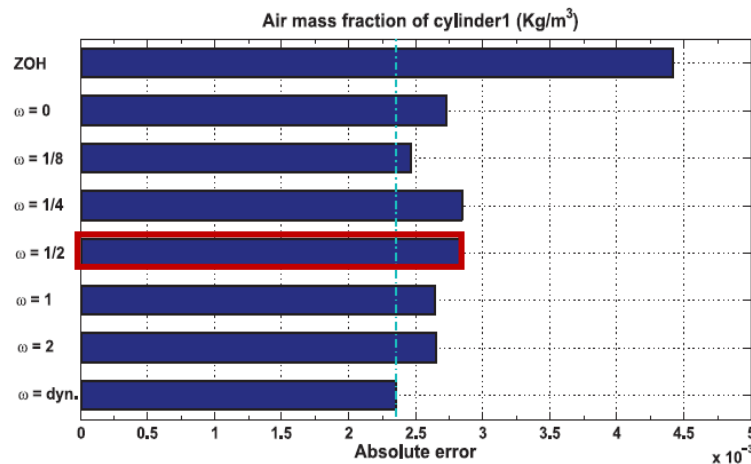
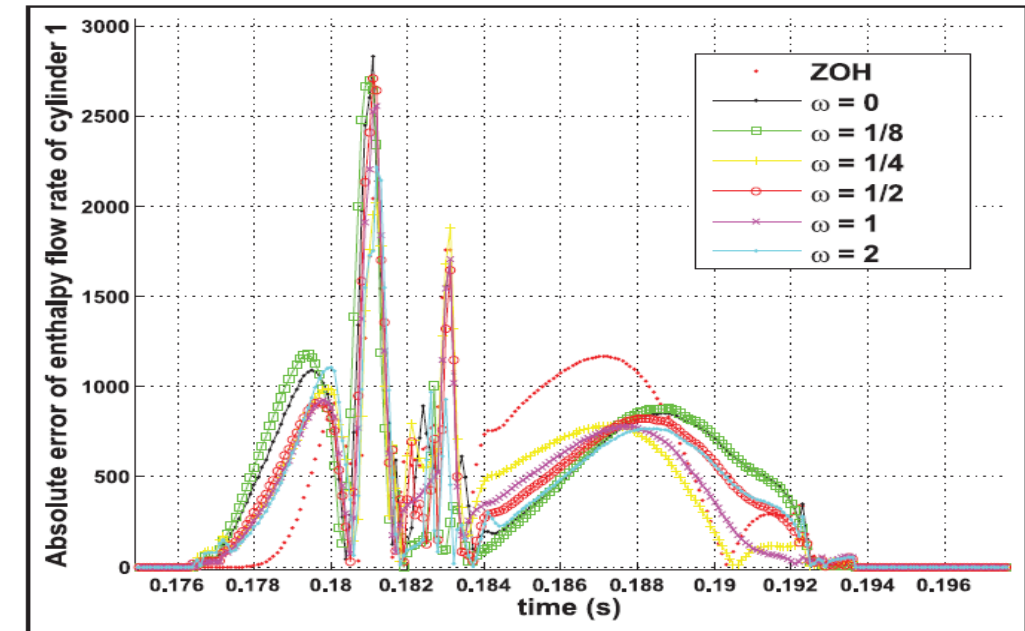
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# SIMULATION RESULTS WITH CHOPtrey

## AUTOMATIC SELECTION OF THE WEIGHTING FACTOR

- No unique best weighting factor  $\omega$  due to complex coupled systems
- ➔ Dynamic selection of  $\omega$ 
  - At each communication step,  $\omega_{\text{best}}$  is selected and used for the current step
  - ➔ Cumulative integration error is the lowest one





## CHOPtrey PERFORMANCE SPEED-UP VERSUS ACCURACY

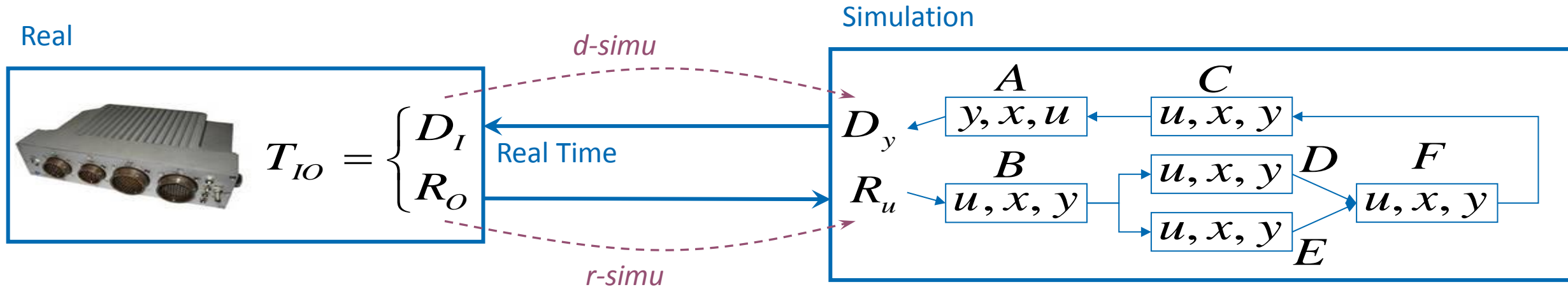
- The speed-up factor is compared with single-threaded reference
- The model is split into 5 threads integrated in parallel on 5 cores
  - Containment of events detection handling → solvers accelerations → overcompensate multi-threading costs
- The relative error variation is compared with ZOH at 100  $\mu$ s

Communication step	Prediction	Speed-up factor	Relative error variation (%)	
			Burned gas density	Fuel density
100 $\mu$ s	ZOH	8.9	-	-
250 $\mu$ s	ZOH	10.01	7	341
	CHOPtrey	10.07	-26	21

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# REAL TIME SIMULATION FROM REAL TIME TO SIMULATED TIME



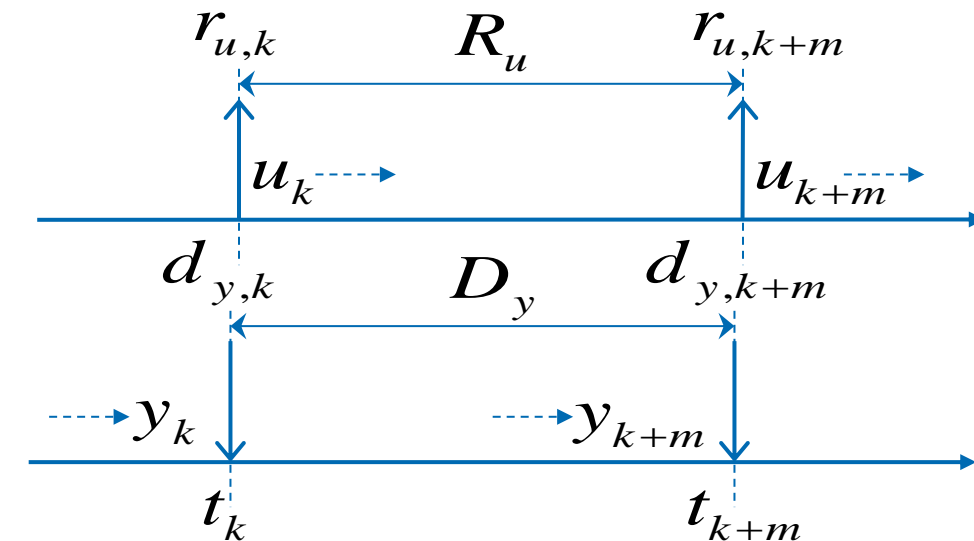
## Relative simulation constraints

At each interaction :  $n \times T_{IO} = t_k$

- $y_k$  required  $\rightarrow$  deadline  $d_{y,k}$
- $u_k$  available  $\rightarrow$  release  $r_{u,k}$

## Impact on the underlying computations

- Linked to the nature of the models and their interconnection
- How to propagate when step time (h) are different for models A, B C, ...?

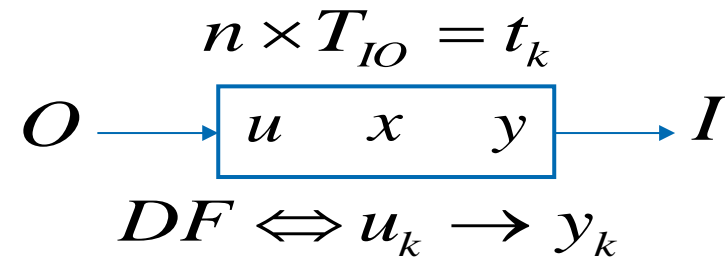


# INTRA-MODEL PROPAGATION AND « DIRECT FEEDTHROUGH »

*Absolute Simu constraints with offset*

$$R_v, D_v = (\Phi, T)$$

offset
period



- Propagation *r-simu*  $R_u = R_O$ 
  - $R_x$ : offset  $h$
  - $DF \Rightarrow R_y = R_u$
  -

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- Propagation *d-simu*  $D_y = D_I$ 
  - $D_x$  : no offset
  - $DF \Rightarrow D_u = D_y$
  -

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*Absolute Simu constraints with offset*

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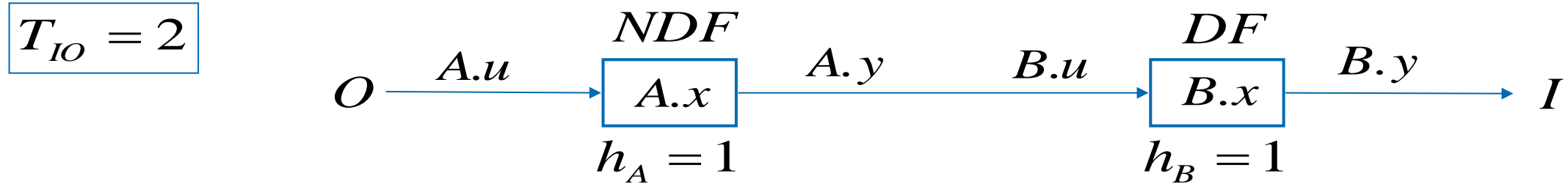
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  - $NDF \Rightarrow R_y$ : offset  $h$

- Propagation *d-simu*  $D_y = D_I$ 
  - $D_x$ : no offset
  - $DF \Rightarrow D_u = D_y$
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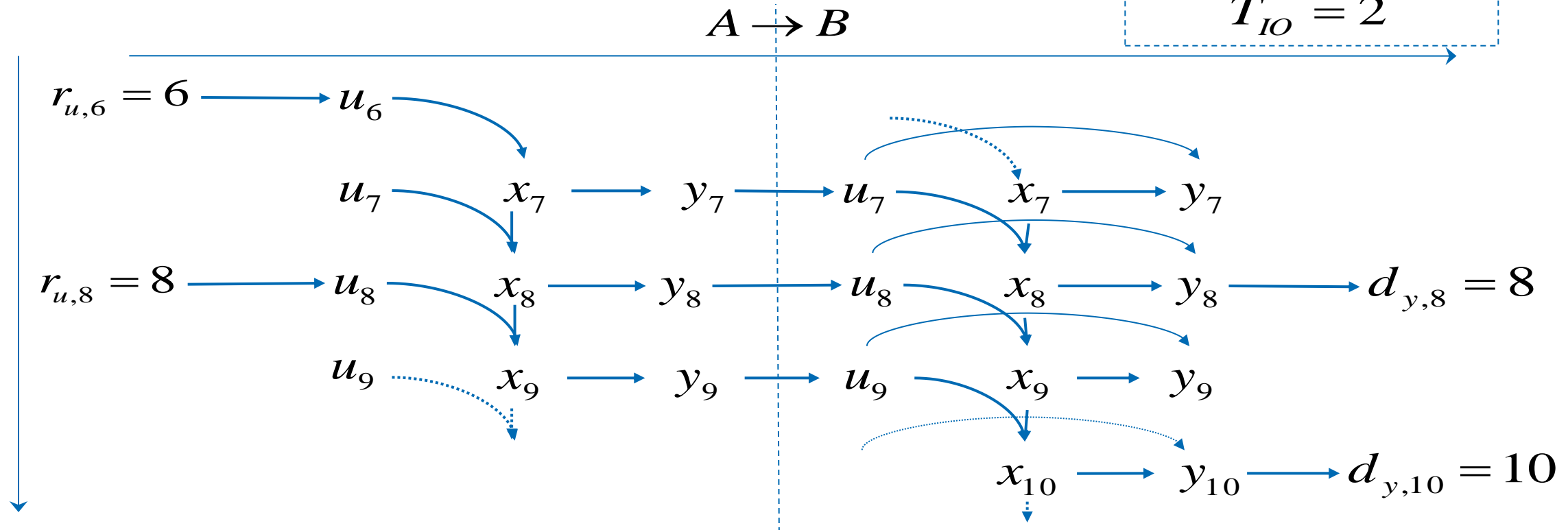
# EXAMPLE OF PROPAGATION RELATIVE MESHES



$(\Phi, T) \backslash v$	$A.u$	$A.x$	$A.y = B.u$	$B.x$	$B.y$
$R_v$	$R_O = (0,2)$	$(1,2)$	$(1,2)$	$(2,2)$	$(1,2)$
$D_v$	$(-1,2)$	$(0,2)$	$(0,2)$	$(0,2)$	$D_I = (0,2)$



# EXAMPLE OF PROPAGATION ABSOLUTE SIMULATION CONSTRAINTS



$$\forall D_v = (\Phi, T), \quad d_{v,i} = \left\lceil \frac{h \times i - \Phi}{T} \right\rceil \times T$$

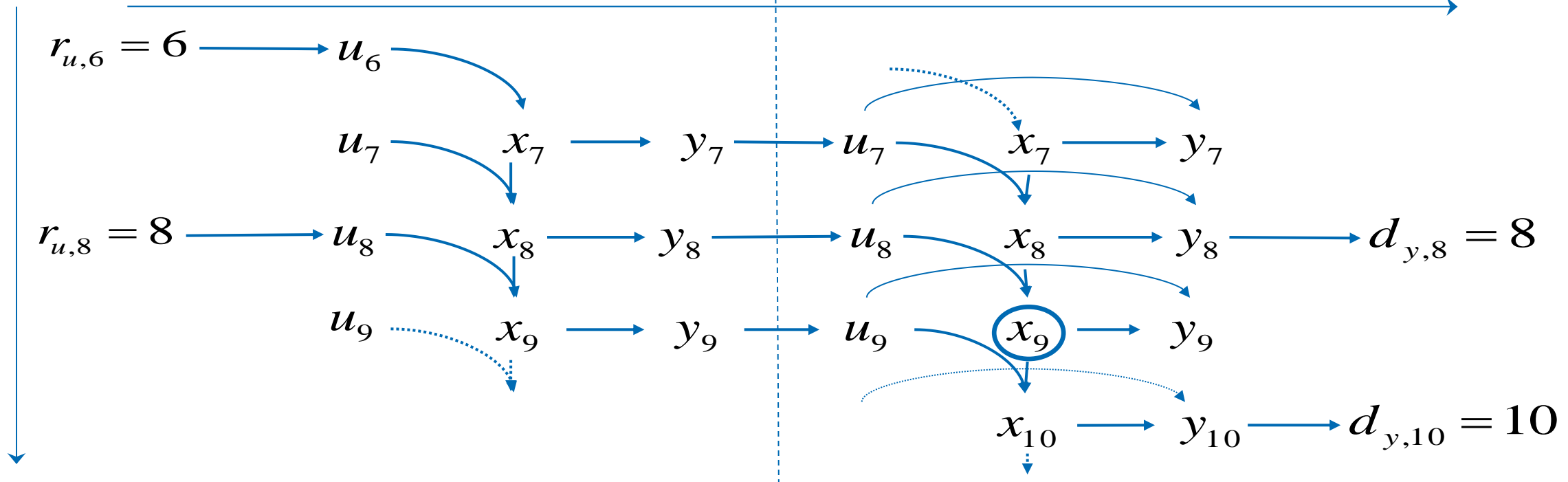
$$\forall R_v = (\Phi, T), \quad r_{v,i} = \left\lfloor \frac{h \times i - \Phi}{T} \right\rfloor \times T$$

# EXAMPLE OF PROPAGATION ABSOLUTE SIMULATION CONSTRAINTS

$$h_A = h_B = 1$$

$$T_{IO} = 2$$

$A \rightarrow B$



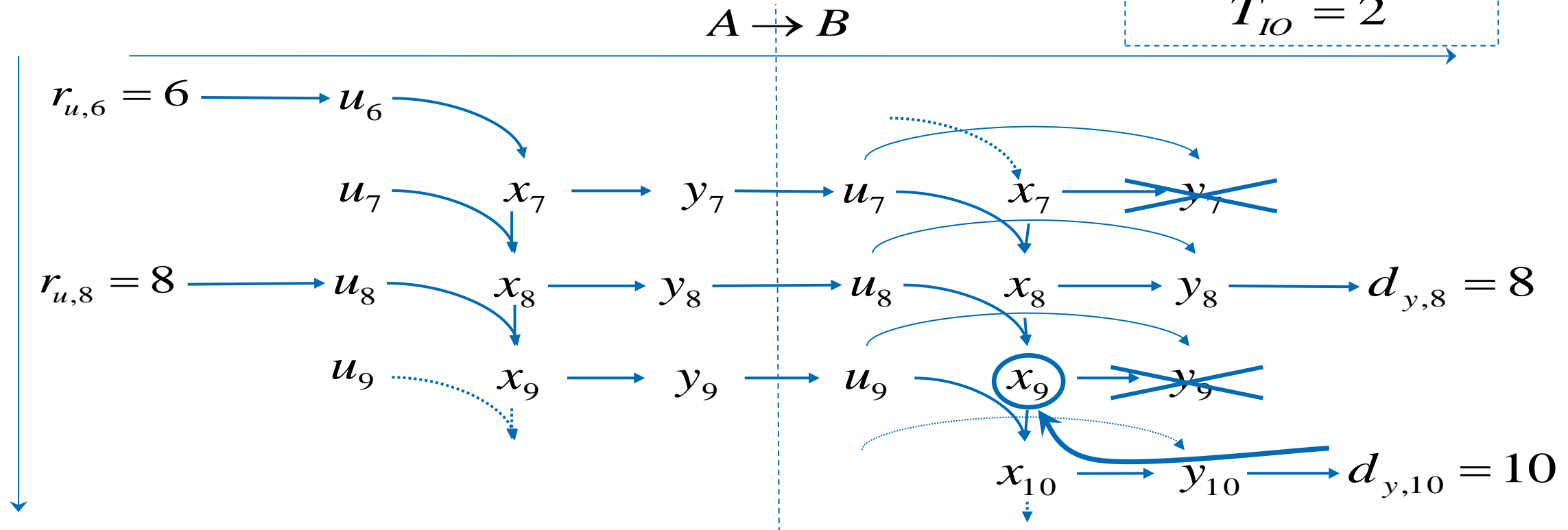
$$\forall D_v = (\Phi, T), d_{v,i} = \left\lceil \frac{h \times i - \Phi}{T} \right\rceil \times T$$

$$\forall R_v = (\Phi, T), r_{v,i} = \left\lfloor \frac{h \times i - \Phi}{T} \right\rfloor \times T$$

	$B.x$
$R_v$	(2,2)
$D_v$	(0,2)

$k = 9$

# EXAMPLE OF PROPAGATION ABSOLUTE SIMULATION CONSTRAINTS



$$\forall D_v = (\Phi, T), d_{v,i} = \left\lceil \frac{h \times i - \Phi}{T} \right\rceil \times T$$

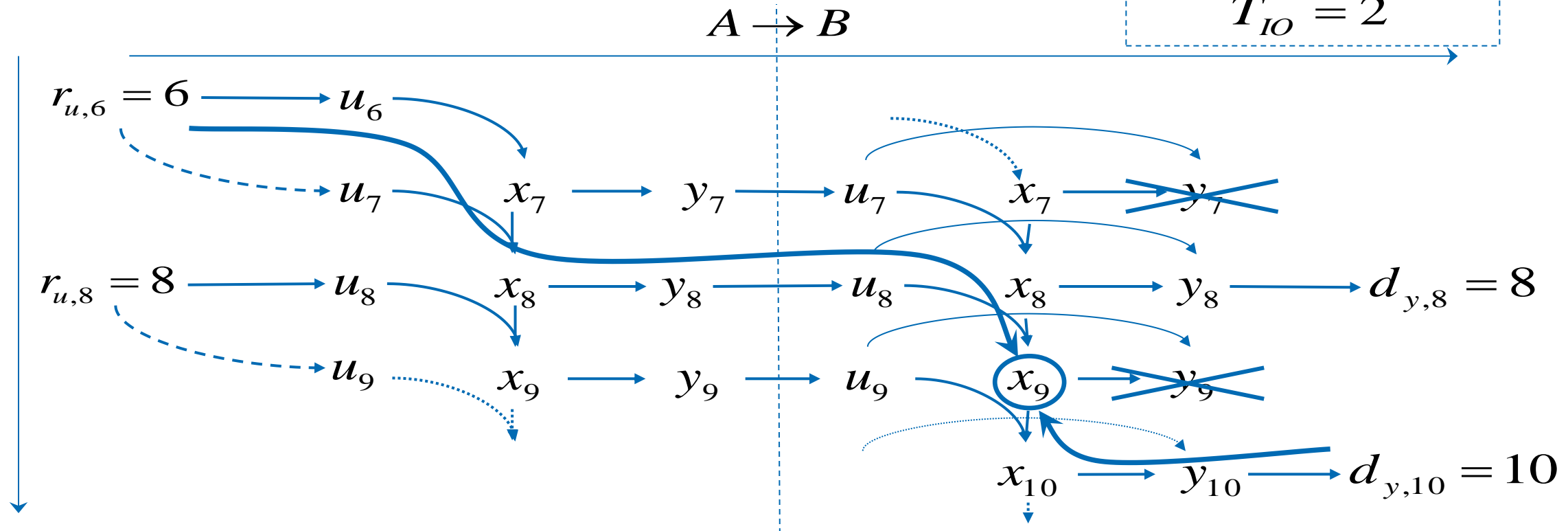
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	$B.x$
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$k = 9$

$$d_{B.x,9} = \left\lceil \frac{1 \times 9 - 0}{2} \right\rceil \times 2 = 10$$

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$$\forall D_v = (\Phi, T), d_{v,i} = \left\lceil \frac{h \times i - \Phi}{T} \right\rceil \times T$$

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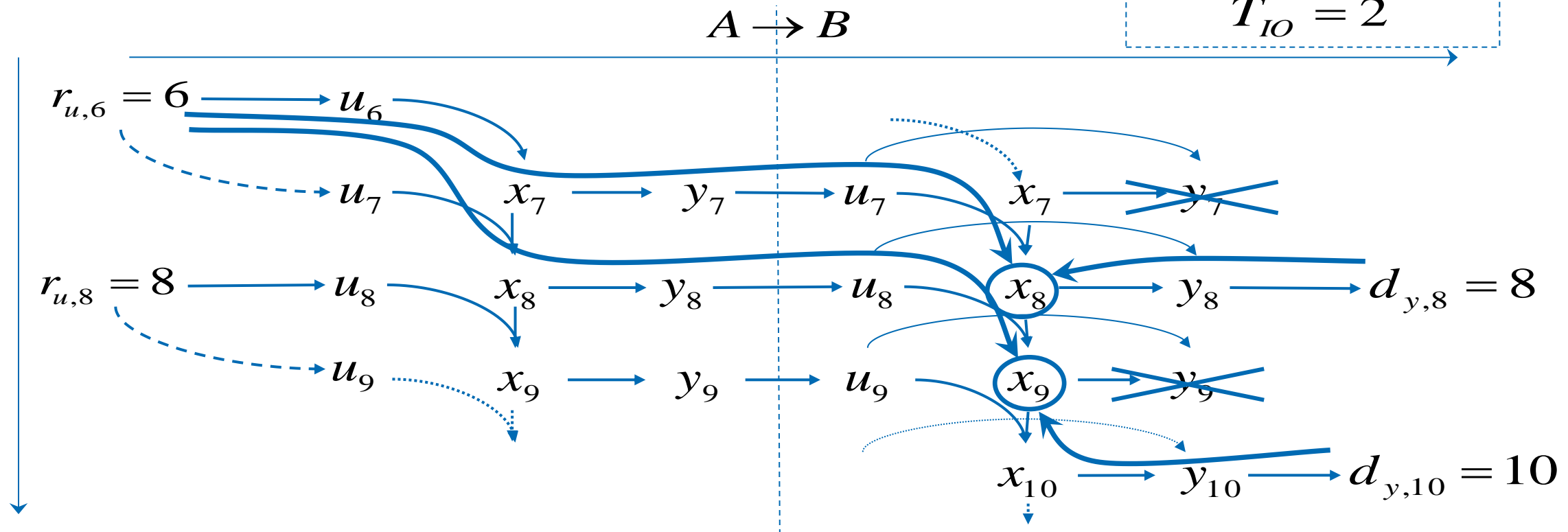
	$B.x$
$R_v$	(2,2)
$D_v$	(0,2)

$k = 9$

$$d_{B.x,9} = \left\lceil \frac{1 \times 9 - 0}{2} \right\rceil \times 2 = 10$$

$$r_{B.x,9} = \left\lfloor \frac{1 \times 9 - 2}{2} \right\rfloor \times 2 = 6$$

# EXAMPLE OF PROPAGATION ABSOLUTE SIMULATION CONSTRAINTS



$$d_{B.x,8} = \left\lceil \frac{1 \times 8 - 0}{2} \right\rceil \times 2 = 8$$

$$r_{B.x,8} = \left\lfloor \frac{1 \times 8 - 2}{2} \right\rfloor \times 2 = 6$$

↕ 2

$$d_{B.x,9} = \left\lceil \frac{1 \times 9 - 0}{2} \right\rceil \times 2 = 10$$

$$r_{B.x,9} = \left\lfloor \frac{1 \times 9 - 2}{2} \right\rfloor \times 2 = 6$$

↕ 4

## FUTURE WORK

- Short term
  - Comparison of RCOSIM heuristics with an exact scheduling algorithm
  - Comparison of RCOSIM offline approach with on-line scheduling
- Mid term
  - Extension of real-time constraints propagation rules to the RCOSIM fine-grained approach
  - Real-time multicore scheduling heuristics for an HiL version of the RCOSIM

## REFERENCES

- S. Saidi et al., “Automatic Parallelization of Multi-Rate FMI-based Co-Simulation on Multi-Core” *Spring Simulation Multiconference* (Apr. 2017)
- A. Ben Khaled-El Feki et al., “CHOPtrey: contextual online polynomial extrapolation for enhanced multi-core co-simulation of complex system”, *Simulation*, 2017, vol. 93(3), pp. 185-200. [DOI : 10.1177/0037549716684026](https://doi.org/10.1177/0037549716684026)
- A. Ben Khaled-El Feki et al., “Fast multi-core co-simulation of Cyber-Physical Systems: application to internal combustion engines”, *Simulation Modelling Practice and Theory*, 2014, vol. 47, pp. 79-91. [DOI : 10.1016/J.SIMPAT.2014.05.002](https://doi.org/10.1016/J.SIMPAT.2014.05.002)