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LOCAL DIRECTIONAL AND UNCOHERENT SEISMIC NOISE FILTERING WITH OVERSAMPLED FILTER BANKS

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Summary

Seismic data are subject to different kinds of unwanted perturbations. These random or organised noises, which can be acquisition or processing related for instance, may disturb geophysical interpretations and thwart attempts at automated processing methods. Since the relative features (*e.g.* amplitude, spectrum) of the signals of interest and the noises may vary locally, signal and noise separation is obtained by a local data-driven filtering with two or three-dimensional oversampled complex filter banks.
Filter banks in general decompose the noisy data onto frequency bands and directions on restricted sub-regions (sub-images or sub-volumes), acting like a local FK with improved properties. The transforms studied in this work present sub-regions smooth overlapping, to avoid tiling effects while allowing signal reconstruction from the transformed domain. The proposed methodology uses limited redundancy filtering that both yield enhanced noise robustness (due to oversampling) and tractable 2D or 3D processing, since they are optimized to limit the redundancy cost.
Coupling those redundant transforms with a processing method designed to detect and compute locally dominant directions, and to remove unwanted directions and random noise, leads to good visual results. Tests were performed on 2D and 3D seismic data.

Introduction

Due to the oscillatory nature of seismic data, the Fourier transform (FT) and its avatars (*e.g.* the $f-k$ filter) are common tools [Yi01] in geosciences for noise removal, data interpolation [ZwGi07], deconvolution, attribute estimation, migration... Basically, the monodimensional discrete FT consists in selecting a number of frequency bins, which may be attenuated or enhanced. In two dimensions or more, dips are converted into peaks or lines in the frequency domain. Recently, other tools such as the wavelet transform (WT) have been proposed to alleviate the lack of local representativeness of the FT. The discrete WT is traditionally implemented using a bank of a low-pass and an high-pass filter, followed by a two-fold subsampling operator, to keep a number of coefficients equal to the number of data samples in the transformed domain. It is now perceived that in strong noise cases, such no-redundant transforms suffer from reconstruction artifacts. Consequently, redundant and oversampled representations generally improve seismic processing. Moreover, the dyadic structure of the WT sometimes lacks in frequency selectivity. Due to their discrete implementation with filters and subsampling operators, FT and WT are special instances of a wider class of signal processing tools called filter banks, which may be designed in general with relatively precise frequency separation, local analysis properties and a certain amount of redundancy. Since seismic data represent huge volumes, we propose a novel use of M-band oversampled complex filter banks (FBs) for local directional and incoherent noise filtering. They possess both the local frequency selectivity desired for local direction estimation and a controllable redundancy to increase their noise robustness. Their results are illustrated on a synthetic image combining different directional features in Figure 2.

In the following, we recall basics of the filter bank theory and provide examples of 2D and 3D noise removal in actual seismic data.

1 Oversampled filter banks

1.1 Polyphase representation

We first recall what a filter bank is and explain briefly the associated mathematical issues. Figure 1 represents a 1D M-band filter bank structure. A 1D signal $(x(n))_{n\in\mathbb{Z}}$ is decomposed using M filters with impulse responses: $(h_i)_{0\le i<M}$, each one having finite length kN with $k\in\mathbb{N}^*$. A decimation by an integer N is then performed. From the lapped transform (LT) viewpoint, we therefore have $k-1$ overlapping blocks of size N, meaning that for a given data block of length N we consider $k-1$ neighbouring blocks. The purpose of a LT is to avoid the blocking or tiling effects that block transforms can produce.
The M outputs of the analysis FB are denoted by $(y_i(n))_{0\le i<M}$. The overall redundancy of the transform is thus $M/N=k'$. We are interested in the oversampled case, *i.e.* $k'>1$. Redundancy is very useful to get more robust processing, but an increased redundancy is tantamount to increased computational burden. So the choice of k' will be the result of a trade off between robustness and computation cost.

The outputs of the analysis FB can then be expressed, for all $i\in\{0,\dots,M-1\}$ and $n\in\mathbb{Z}$, as

$$y_i(n)=\sum_p h_i(p)x(Nn-p)=\sum_\ell\sum_{j=0}^{N-1}h_i(N\ell+j)x(N(n-\ell)-j). \tag{1}$$

Let $\boldsymbol{H}(\ell)=(h_i(N\ell+j))_{0\le i<M,0\le j<N}$, $\ell\in\{0,\cdots,k-1\}$ be the k matrices obtained from the impulse responses of the filters. We also define the polyphase vector signal from the input signal $x(n)$: $\forall n\in\mathbb{Z}$, $\mathbf{x}(n)=(x(Nn-j))_{0\le j<N}$. A more concise form for Eq. 1 is:

$$\mathbf{y}(n)=(y_0(n),\dots,y_{M-1}(n))^\top=\sum_\ell\boldsymbol{H}(\ell)\mathbf{x}(n-\ell)=(\boldsymbol{H}*\mathbf{x})(n),$$

or, equivalently, $\mathbf{y}[z]=\boldsymbol{H}[z]\mathbf{x}[z]$, where $\boldsymbol{H}[z]=\sum_{\ell=0}^{k-1}\boldsymbol{H}(\ell)z^{-\ell}$ is the $M\times N$ polyphase transfer matrix of the analysis filter bank and $\mathbf{x}[z]$ and $\mathbf{y}[z]$ are the z-transforms of $(\mathbf{x}(n))_{n\in\mathbb{Z}}$ and $(\mathbf{y}(n))_{n\in\mathbb{Z}}$, respectively. Similarly, we define the polyphase transfer matrix of the synthesis filter bank: $\widetilde{\boldsymbol{H}}[z]=\sum_\ell\widetilde{\boldsymbol{H}}(\ell)z^{-\ell}$ which is such that $\widetilde{\mathbf{x}}[z]=\widetilde{\boldsymbol{H}}[z]\mathbf{y}[z]$.
The polyphase vector of $(\widetilde{\mathbf{x}}(n))_{n\in\mathbb{Z}}$ is defined similarly as $(\mathbf{x}(n))_{n\in\mathbb{Z}}$ and $\widetilde{\boldsymbol{H}}(\ell)=\left(\widetilde{h}_j(N\ell-i)\right)_{0\le i<N,0\le j<M}$,

$\ell \in \mathbb{Z}$. In other words, if we know the polyphase matrix $\widetilde{\boldsymbol{H}}[z]$, then the synthesis FB is well defined.

1.2 Synthesis filter bank

Usually, when working with filter banks, an analysis filter bank is chosen because of some desirable properties (frequency selection for instance) and then the problem is to build a synthesis filter bank achieving perfect reconstruction property (*i.e.* the output of the synthesis FB is exactly the same as the input of the analysis FB). With the previous notations, it is necessary and sufficient to prove that a polyphase matrix $\widetilde{\boldsymbol{H}}[z]$ such that: $\widetilde{\boldsymbol{H}}[z]\boldsymbol{H}[z] = \boldsymbol{I}_N$ exists, to prove that the problem is invertible.

In a previous work [GaDu06], we have proposed an algorithm to check whether an analysis FB is invertible. We also described a method, based on solving linear system, to compute *one* synthesis FB denoted $\widetilde{\boldsymbol{H}}_0$. The fact that we are considering a redundant filter bank implies that the problem is underdetermined, *i.e.* a whole class of inverse FB exists. An optimization step [GaDu07] can then be used to exploit those degrees of freedom by choosing a synthesis FB with good time/space localization. For instance the following function:

$$J(\widetilde{h}) = \sum_{j=0}^{M-1} \frac{\sum_m (m-\overline{m}_j)^2 \left|\widetilde{h}_j(m)\right|^2}{\sum_m \left|\widetilde{h}_j(m)\right|^2}$$

could be optimized under the linear constraint defining the inverse filter bank $\widetilde{\boldsymbol{H}}_0$.

2 Directional filtering

2.1 Direction and disorder estimation

It is a well known fact that the vector (a,b) represents the orthogonal direction to the visually dominant one in a synthetic image, such as $\sin(ax+by)$ in 2D. The locally stratified structure of seismic data induces layered and highly anisotropic images close to those synthetic images. In order to estimate an orientation, it is sufficient to evaluate the vector: (a,b). By studying the magnitude of the multidimensional Fourier Transform, two symmetric peaks appear, thus defining an orientation. This direction is supported by a vector approximating (a,b). It is interesting to note that a real frequency transform (such as the Discrete Cosine Transform) does not catch the orientation information in a clear enough method, making it impossible to separate features oriented with an angle θ from features with an angle $-\theta$.

Let p_1 be the value of the maximum magnitude peaks. p_1 represents the *weight* of the dominant orientation. If there exists a second direction with a lesser importance than the first, it is clear that the magnitude image of the Fourier Transform will present secondary symmetric peaks, representing the secondary orientation. The value p_2 of those peaks will be called the weight of the secondary orientation.

In this paper, an area will be called *disordered*, if it possesses several orientations of similar weight. With the previous notations, we define a *disorder estimation* as:

$$D_{est} = 1 - \frac{p_2}{p_1}.$$

It is clear that if the two directions have similar weights, p_1 and p_2 will be close and then D_{est} will be near 0. Whereas if the two weights are very different (and since, by definition, $p_2 < p_1$) then D_{est} will be close to 1. Other functions based on the values p_1 and p_2 could also be used. The previous explanations were proposed in the 2D case, easier to formulate, but it can be extended naturally to 3D.

To illustrate this direction detection method, we show on Figure 2 several examples of synthetic images and their associated transform in the frequency domain. When only one direction is displayed, we see two symmetric peaks (with respect to the center of the image), and if two orientations are present, then there are two couples of peaks with weights linked to the relative importance of each direction.

2.2 Filtering

Our purpose is to perform directional filtering on seismic data. The filtering method used in this work is based on the identification of the dominant direction and then removal of all other unwanted orientations.

To avoid over-filtering of disordered areas, where several directions of interest can be found, we propose to evaluate D_{est} and adapt our filtering strategy according to its value. It is clear that, if there are two different directions with similar importance then any result regarding the dominant orientation will be very doubtful. In this sense, the disorder attribute can also be viewed as a confidence level in the computation of the primary direction.
The processing takes place in the Fourier domain after each local transform and presents the following steps:

① Computation of the primary orientation: symmetric maximum peaks are searched in the magnitude image obtained from the transform, and their direction is computed. The value p_1 is also kept.

② Computation of the secondary orientation: with the primary direction known, the secondary one can be found by searching the magnitude image (in parts not corresponding to the primary direction) for symmetric maximum peaks, giving the value p_2 and the secondary orientation.

③ Computation of D_{est}. If D_{est} is close to 1, then we keep only the primary direction, but if the attribute is smaller then we keep a mixture of the two dominant directions.

④ Hard-thresholding to remove small coefficients likely to represent noise. The threshold is computed by taking a fraction of p_1.

The third step of this method proposes to use a mixture of primary and secondary orientations if D_{est} is not close enough to 1. The mixture that was used in this work was to give a 1 weight to the coefficient corresponding to the primary direction and $1 - D_{att}$ for the secondary.

2.3 Transform used in this work

The analysis filters used here are a windowed generalized Fourier transform derived from [YoKi93] and expressed as: $h_i(n) = \boldsymbol{E}(i,n)h_a(n)$, where

$$\boldsymbol{E}(i,n) = \frac{1}{\sqrt{k'N}} e^{-\imath (i - \frac{k'N}{2} + \frac{1}{2})(n - \frac{kN}{2} + \frac{1}{2}) \frac{2\pi}{k'N}},$$

and $(h_a(n))_{1 \leq n \leq kN}$ is a non vanishing analysis window such as: $h_a(n) = \sin\left(\frac{n\pi}{kN+1}\right)$. This transform is complex, in accordance with our remark in section 2.1. By applying the above monodimensional transform separably in all directions we define the multi-dimensional transform we practically use on seismic data.

3 Results

We have applied the filtering method on 2D and 3D seismic data using the following parameters: $N = 16$ (decimation factor), $k = 3$ (overlapping factor) and $k' = 7/4$ (redundancy).
On Figure 3, a sample of 3D seismic data and the result of the filtering are proposed. We can see that important underlying structures are enhanced by this filtering. To see better the direction selection, we propose to look, on figure 4, at some local 2D samples before and after filtering. We can see that the unwanted directions were completely removed, while significant structures were well preserved and smoother (leading to easier detection or horizon picking).

Conclusion

In this paper we have presented filter banks and elements about their choices and construction. We have explained how to perform directional filtering on 2D or 3D seismic data using oversampled and overlapping filter banks, and applied them on real seismic data leading to good visual results.

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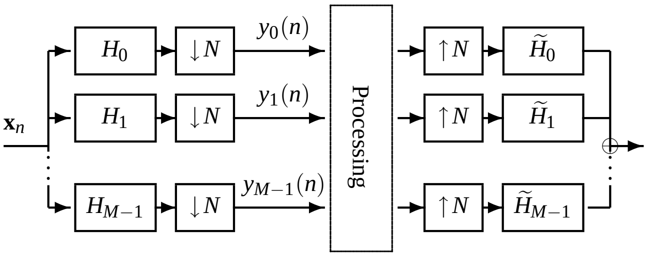

Figure 1: Oversampled *M*-channel filter banks.

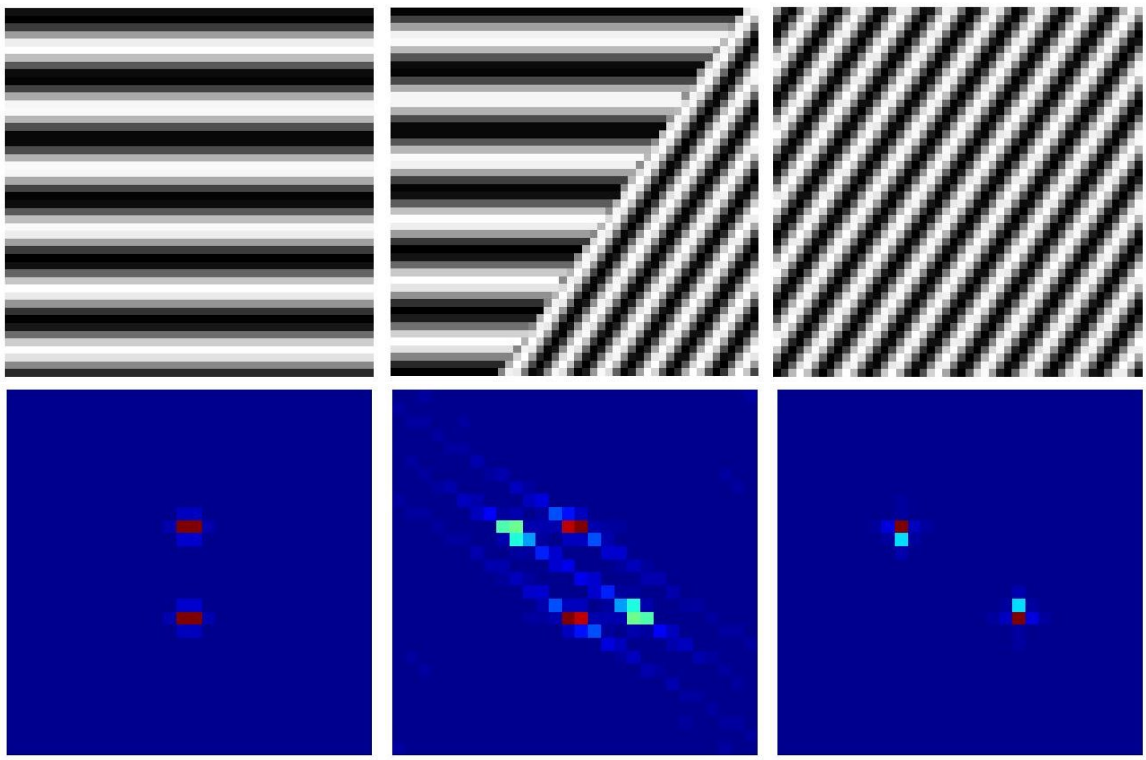

Figure 2: Synthetic images and their associated transform.

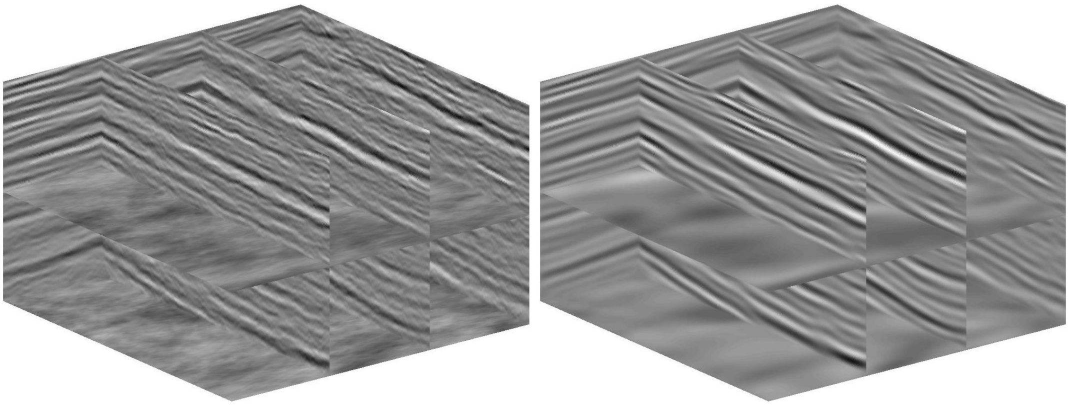

Figure 3: A sample of 3D seismic data before and after using the filtering method.

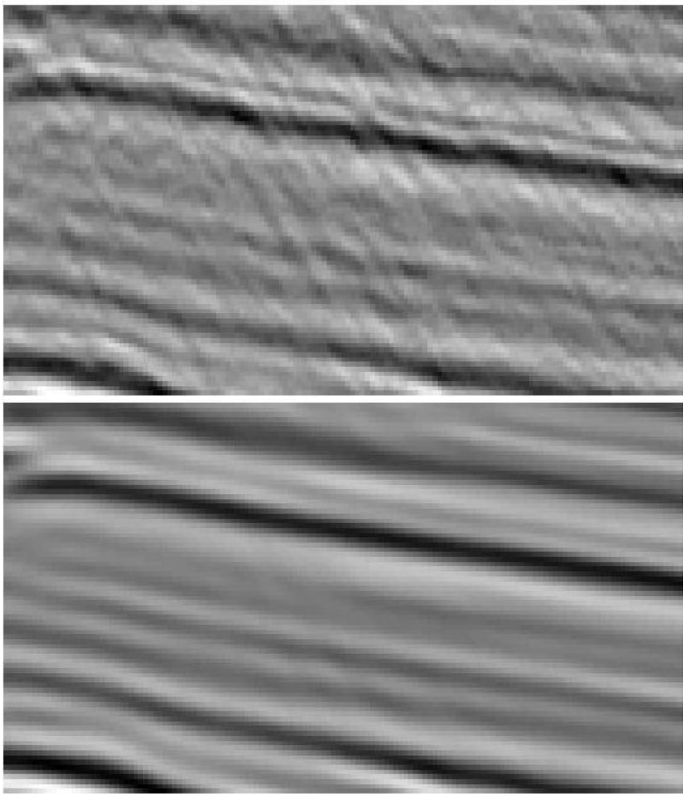

Figure 4: A sample of 2D seismic data before and after using the filtering method.