

Examples of counterexamples: a book conspectus in mathematics, computer science, philosophy...

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Counterexample: any exception to a generalization. Counterexamples are often used in science (and philosophy), as a means to setting boundaries. In mathematics at large, well-chosen counterexamples may bound possible theorems, disprove certain conjectures. Recent breakthroughs show that artificial intelligence tools like LLMs with proof assistants are becoming able to generate non-trivial counterexamples, disproving long-standing math conjectures. This challenges human assumptions about machine reasoning limitations. This conspectus means to gather and share counterexample book references in mathematics (on algebra, analysis, computer science, calculus, logic, philosophy, probability, statistics, topology) and related topics (computer science, philosophy). Latest version: 10.5281/zenodo.14740765.

Keywords— Artificial intelligence, Beispiel, Beweis , computer science, contradiction, contre-exemple, counterexample, example, exemple, Gegenbeispiel, informatique, intelligence artificielle, LLM, mathematics, Mathematik, mathématiques, preuve, proof, refutation, réfutation

Irrefutability is not a virtue of a theory (as people often think) but a vice. Karl R. Popper



Revision	Date	Author(s)	Description
v1.0.0	2025-01-14	Laurent	First release
v1.0.1	2025-01-26	Laurent	Posted to Zenodo. Added versioning. Obtained a DOI
v1.0.2	2025-02-12	Laurent	Slightly changed the formatting
v1.0.3	2026-08-09	Laurent	Added computer science references, thumbnails and known links to books

1 Impetus, motivation, credits

X-Twitter post “Examples of books on counterexamples”¹ (May 25th, 2023) was triggered by Shen Comix’s cartoon in Figure 1, referring to Borwein–Borwein integrals: a series of sums of products of cardinal sines (sinc), whose eight-first terms share the same $\frac{\pi}{2}$ value, while the ninth term is subtly different [BB01].



Figure 1: Shen Comix: detecting patterns in series, referring to Borwein–Borwein sinc integrals.

Counterexamples are important in science — notably to invalidate arguments, refine conjectures, develop theories, enhance education. Recently, artificial intelligence programs with prof assistants (Claude Fable/Mythos, ChatGPT Sol, Lean) are proving able to assist humans in generating breakthrough counterexamples, disproving long-standing math conjectures. This impetus motivated this conspectus² of books (with the associated L^AT_EXbib file) focused on counterexamples (with one exception), on algebra, (complex and real) analysis, calculus, control, differential equations, functional analysis, graph theory, logic, mathematical programming, measure theory, model checking, operator theory, philosophy, probability, statistics, topology. Occasionally, one may find a book chapter when no book seems to exist. Publications on counterexamples are numerous (for instance: “Examples and Counterexamples” journal³). This document will be amended when additional book-like references are available. Thanks to X users: @GrdCrif, @datagenproc, @latzplacian, @sjaak_blom, @LGcommaI, @eremeticus.

References

- [App09] Jürgen Appell. *Analysis in Beispielen und Gegenbeispielen*. Springer-Lehrbuch. Springer, Berlin, Heidelberg, 2009.
- [App21] Jürgen Appell. *Analysis in Beispielen und Gegenbeispielen. Eine Einführung in die Theorie reeller Funktionen*. Springer-Lehrbuch. Springer, Berlin, Heidelberg, 2nd edition, 2021.
- [BB01] David Borwein and Jonathan M. Borwein. Some remarkable properties of sinc and related integrals. *Ramanujan J.*, 5(1):73–89, 2001.
- [BB15] Andrei Bourchtein and Ludmila Bourchtein. *CounterExamples. From elementary calculus to the beginnings of analysis*. CRC Press, 2015.
- [BV10] Jonathan M. Borwein and Jon D. Venderwerff. *Convex functions. Constructions, characterizations and counterexamples*. Number 109 in Encyclopedia of Mathematics and its Applications Book. Cambridge University Press, 1st edition, 2010.
- [CM78] Michael Capobianco and John C. Molluzzo. *Examples and counterexamples in graph theory*. North-Holland, 1978.
- [CV03] Edmund Clarke and Helmut Veith. *Counterexamples Revisited: Principles, Algorithms, Applications*, pages 208–224. Springer Berlin Heidelberg, 2003.
- [FS07] John Erik Fornæss and Berit Stensønes. *Lectures on counterexamples in several complex variables*. AMS Chelsea Publishing, 2007.
- [GO90] Bernard R. Gelbaum and John Meigs Hubbell Olmsted. *Theorems and counterexamples in mathematics*. Problem Books in Mathematics. Springer, 1st edition, 1990.
- [GO03] Bernard R. Gelbaum and John Meigs Hubbell Olmsted. *Counterexamples in analysis*. Dover Publications, 2003.
- [Gre09] Harvey J. Greenberg. *Myths and Counterexamples in Mathematical Programming*. 2009.
- [Hau07] Bertrand Hauchecorne. *Les contre-exemples en mathématiques*. Ellipses, 2nd edition, 2007.
- [Hut81] Harry C. Hutchins. *Examples of commutative rings*. Polygonal Publishing House, 1981. See also: [Hut18].

¹Tweet “Examples of books on counterexamples” x.com/laurentduval/status/1661725617891450880

²Inspired by the drone tune: Raison d’être, Conspectus, 1994.

³<https://www.sciencedirect.com/journal/examples-and-counterexamples/>

- [Hut18] Harry C. Hutchins. *Correction sheet for “Examples of commutative rings”*. Polygonal Pub House, 2018.
- [Kha82] S. M. Khaleelulla. *Counterexamples in topological vector spaces*. Number 936 in Lecture Notes in Mathematics. Springer, 1982.
- [Kly10] Sergiy Klymchuk. *Counterexamples in calculus*. Mathematical Association of America, 2010.
- [Kum11] Bernd Kummer. Counterexamples in optimization and non-smooth analysis. Vorlesung April-July 2011, 2011.
- [Lak76] Imre Lakatos. *Proofs and refutations. The Logic of Mathematical Discovery*. Cambridge University Press, 1976.
- [LvF72] Karel Lambert and Bas C. van Fraassen. *Derivation and counterexample: an introduction to philosophical logic*. Dickenson Publishing, 1972.
- [MK09] John Mason and Sergiy Klymchuk. *Using counter-examples in calculus*. Imperial College Press, 2009.
- [Mor22] Mohammed Hichem Mortad. *Counterexamples in operator theory*. Springer, 2022.
- [Piu25] Alexei B. Piunovskij. *Counterexamples in Markov decision processes*. Number vol. 6 in Series on optimization and its applications. World Scientific Publishing, New Jersey, 2025. Includes bibliographical references and index.
- [Pop00] Karl R. Popper. *Conjectures and refutations. The growth of scientific knowledge*. Routledge, London [u.a.], 5. ed. (rev.), reprint edition, 2000. Includes bibliographical references and indexes.
- [Pop07] Karl R. Popper. *Conjectures and refutations*. Routledge Classics. Routledge, London [u.a.], repr. edition, 2007.
- [Ras91] John M. Rassias. *Counter examples in differential equations and related topics*. Word scientific, 1991.
- [RB11] A. R. Rajwade and A. K. Bhandari. *Surprises and counterexamples in real function theory*. Number 42 in Textbooks and reading in mathematics. Hindustan Book Agency, 2011.
- [RS86] Joseph P. Romano and Andrew F. Siegel. *Counterexamples in probability and statistics*. Chapman & Hall/CRC, 1986.
- [Ser03] Semen Ya. Serovaiskii. *Counterexamples in optimal control theory*. Inverse and ill-posed problems. De Gruyter, 2003.
- [SK21] René L. Schilling and Franziska Kühn. *Counterexamples in measure and integration*. University of Cambridge, 2021.
- [SS78] Lynn Arthur Steen and J. Arthur Seebach, Jr. *Counterexamples in topology*. Springer, 2nd edition, 1978.
- [Sto13] Jordan M. Stoyanov. *Counterexamples in probability*. Dover Publications, 3rd edition, 2013.
- [WH93] Gary L. Wise and Eric B. Hall. *Counterexamples in probability and real analysis*. Oxford University Press, 1993.
- [Wri14] Rachel Wright. *Theorems And Counterexamples In Algebraic Rings Theory*. Auris Press, 2014. (elusive reference).
- [ZC19] Xiaoxiang Zhang and Jianlong Chen. *Counterexamples in ring theory*. Science Press, 2019.