On echoes and morphing
Adaptive filtering in the complex wavelet domain with unary filters: application to multiple suppression in geophysics

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On echoes and morphing

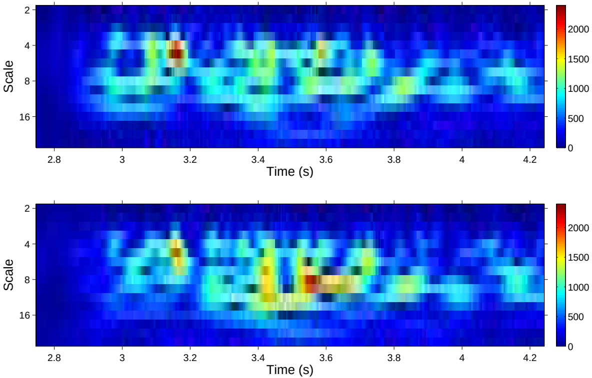

Figure 1: ... and adaptive subtraction

Agenda

1. Issues in geophysical signal processing
2. Problem: multiple reflections (echoes)
   - adaptive filtering with approximate models
3. Complex, continuous wavelets
   - and how they (may) simplify adaptive filtering
4. Discretization, redundancy and unary filters (morphing)
   - being practical: back to the discrete world
5. Results
6. Conclusion

Issues in geophysical signal processing

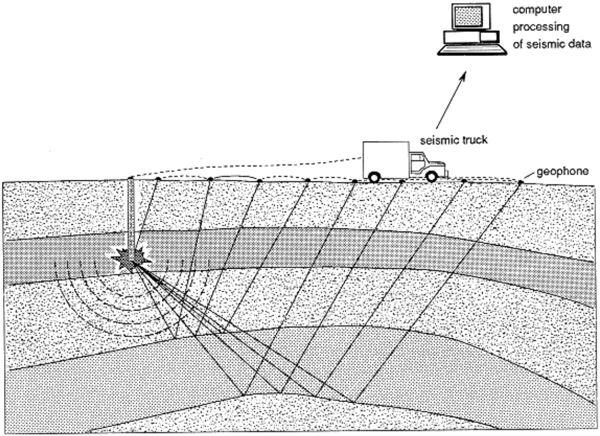

Figure 2: Seismic data acquisition and wave fields.

Issues in geophysical signal processing

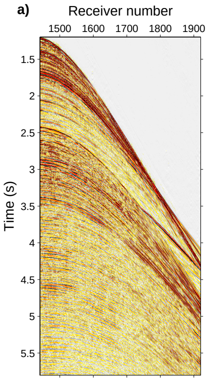

Figure 3: Seismic data: aspect & dimensions (time, offset)

Issues in geophysical signal processing

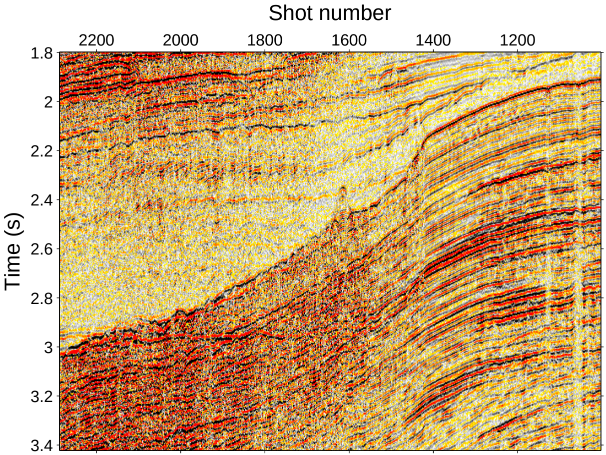

Figure 4: Seismic data: aspect & dimensions (time, offset)

Issues in geophysical signal processing

Reflection seismology:

- seismic waves propagate through the subsurface medium
- seismic traces: seismic wave fields recorded at the surface
  - primary reflections: geological interfaces
  - many types of distortions/disturbances
- processing goal: extract relevant information for seismic data
- led to important signal processing tools:
  - ℓ_1-promoted deconvolution (Claerbout, 1973)
  - wavelets (Morlet, 1975)
- exabytes (10^6 gigabytes) of incoming data
  - need for fast, scalable algorithms

Multiple reflections and models

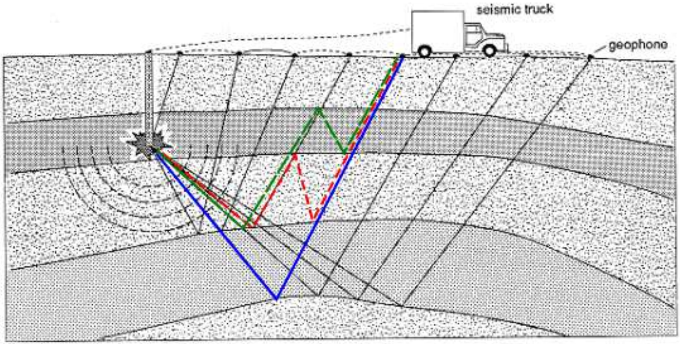

Figure 5: Seismic data acquisition: focus on multiple reflections

Multiple reflections and models

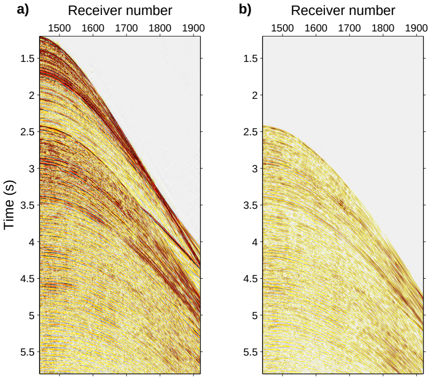

Figure 5: Reflection data: shot gather and model

Multiple reflections and models

Multiple reflections:

- seismic waves bouncing between layers
- one of the most severe types of interferences
- obscure deep reflection layers
- high cross-correlation between primaries (p) and multiples (m)
- additional incoherent noise (n)
- $d(t) = p(t) + m(t) + n(t)$
  - model-based multiple attenuation: $x_1(t)$, $x_2(t)$, $x_3(t)$
- how to use approximate models?

Multiple reflections and models

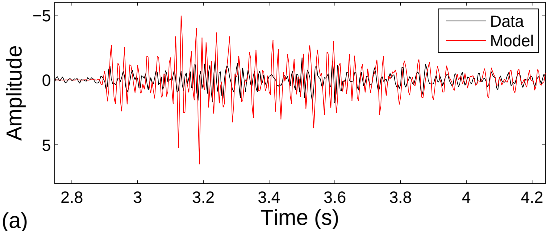

Figure 6: Multiple reflections: data trace d and model x_1

Multiple reflections and models

Multiple filtering:

- multiple prediction (correlation, wave equation) has limitations
- models are not accurate
  - $m(t) \approx a_k(t) x_k(t - \tau_k(t))$?
  - standard: identify, apply a matching filer, subtract
- primaries and multiples are not (fully) uncorrelated
  - same (seismic) source
  - similarities/dissimilarities in time
  - similarities/dissimilarities in frequency
- variations in amplitude, waveform, delay
- issues in matching filter length:
  - short filters and windows: local details
  - long filters and windows: large scale effects

Multiple reflections and models

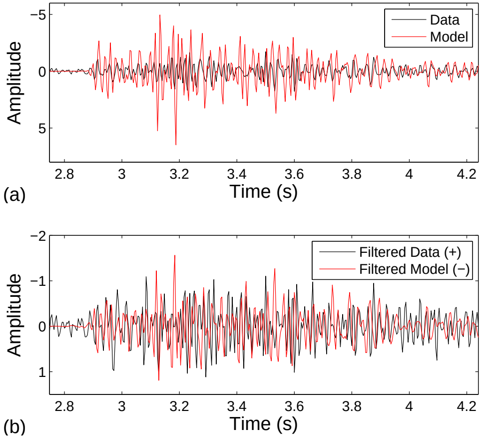

Figure 7: Multiple reflections: data trace, model and adaptation

Multiple reflections and models

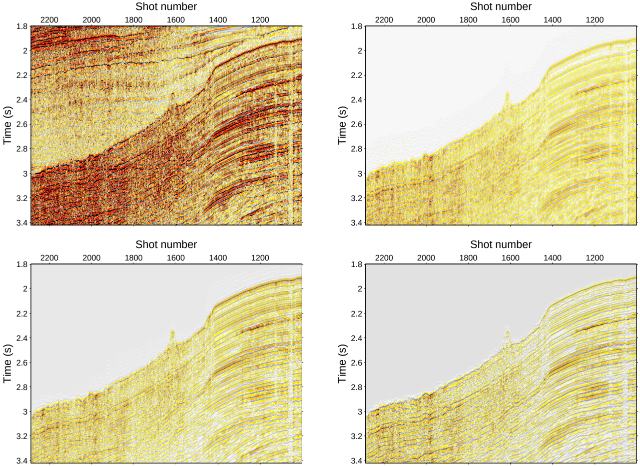

Figure 8: Multiple reflections: data trace and models, 2D version

Multiple reflections and models

- A long history of multiple filtering methods
  - general idea: combine adaptive filtering and transforms
    - data transforms: Fourier, Radon
    - enhance the differences between primaries, multiples and noise
    - reinforce the adaptive filtering capacity
  - intrication with adaptive filtering?
    - might be complicated (think about inverse transform)
- Main idea here:
  - exploit the non-stationary in the data
  - naturally allow both large scale & local detail matching
  - work in a complex domain: amplitude and phase representation
  - emulate an analytic signal representation (Hilbert transform)

$\Rightarrow$ Complex, continuous wavelets

- intermediate complexity in the transform
- hyper-simplicity in the (unary) adaptive filtering

Hilbert pairs

Reminders [Gabor-1946][Ville-1948]

$$\widehat{\mathcal{H}\{f\}}(\omega) = -\imath \operatorname{sign}(\omega)\widehat{f}(\omega)$$

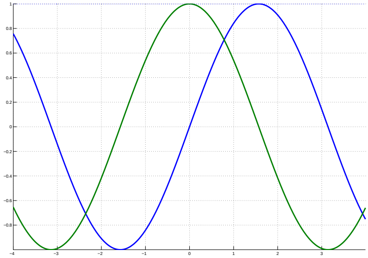

Figure 9: Hilbert pair 1

Hilbert pairs

Reminders [Gabor-1946][Ville-1948]

$$\widehat{\mathcal{H}\{f\}}(\omega) = -\imath\,\mathrm{sign}(\omega)\,\widehat{f}(\omega)$$

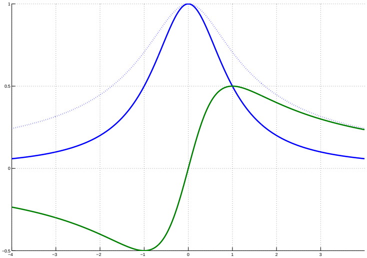

Figure 9: Hilbert pair 2

Hilbert pairs

Reminders [Gabor-1946][Ville-1948]

$$\widehat{\mathcal{H}\{f\}}(\omega) = -\imath \operatorname{sign}(\omega) \widehat{f}(\omega)$$

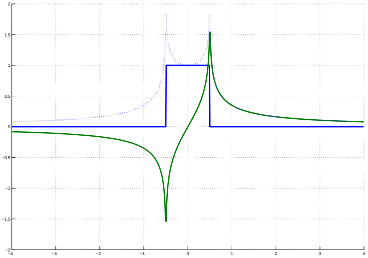

Figure 9: Hilbert pair 3

Hilbert pairs

Reminders [Gabor-1946][Ville-1948]

$$\widehat{\mathcal{H}\{f\}}(\omega) = -\imath\,\mathrm{sign}(\omega)\widehat{f}(\omega)$$

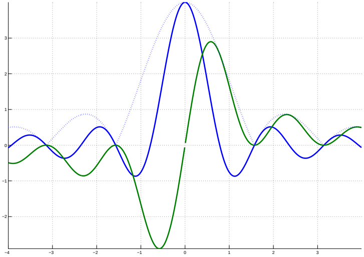

Figure 9: Hilbert pair 4

Continuous wavelets

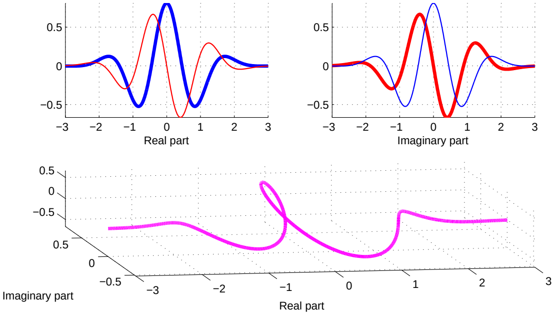

Figure 10: Complex wavelets at two different scales - 1

Continuous wavelets

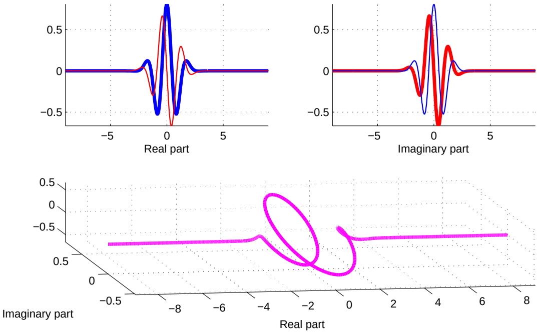

Figure 11: Complex wavelets at two different scales - 2

Continuous wavelets

- Transformation group:

$$\text{affine} = \text{translation } (\tau) + \text{dilation } (a)$$

- Basis functions:

$$\psi_{\tau,a}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-\tau}{a}\right)$$

  - $a > 1$: dilation
  - $a < 1$: contraction
  - $1/\sqrt{a}$: energy normalization
  - multiresolution (vs monoresolution in STFT)

$$\psi_{\tau,a}(t) \xrightarrow{\text{FT}} \sqrt{a}\Psi(af)e^{-\imath 2\pi f\tau}$$

Continuous wavelets

- Definition

$$C_s(\tau, a) = \int s(t)\psi^*_{\tau,a}(t)dt$$

- Vector interpretation

$$C_s(\tau, a) = \langle s(t), \psi_{\tau,a}(t)\rangle$$

projection onto time-scale atoms (vs time-frequency)

- Redundant transform: $\tau \to \tau \times a$ "samples"
- Parseval-like formula

$$C_s(\tau, a) = \langle X(f), \Psi_{\tau,a}(f)\rangle$$

$\Rightarrow$ time-scale domain operations! (cf. Fourier)

Continuous wavelets

Introductory example

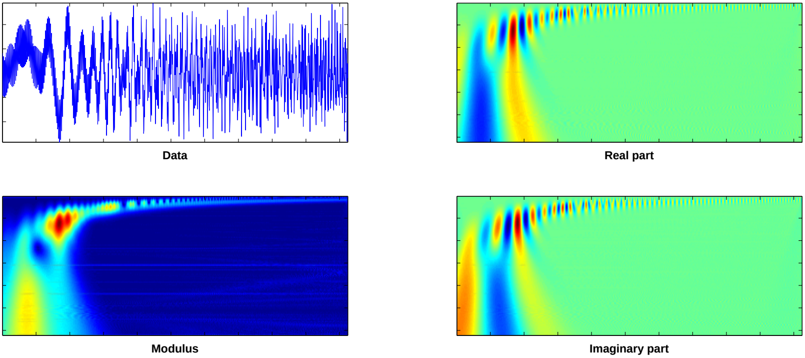

Figure 12: Noisy chirp mixture in time-scale & sampling

Continuous wavelets

Noise spread & feature simplification

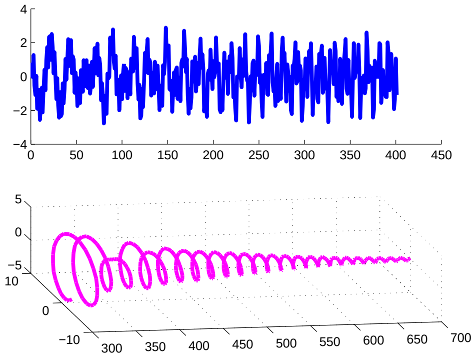

Figure 13: Noisy chirp mixture in time-scale: scale, zoomed wiggle

Continuous wavelets

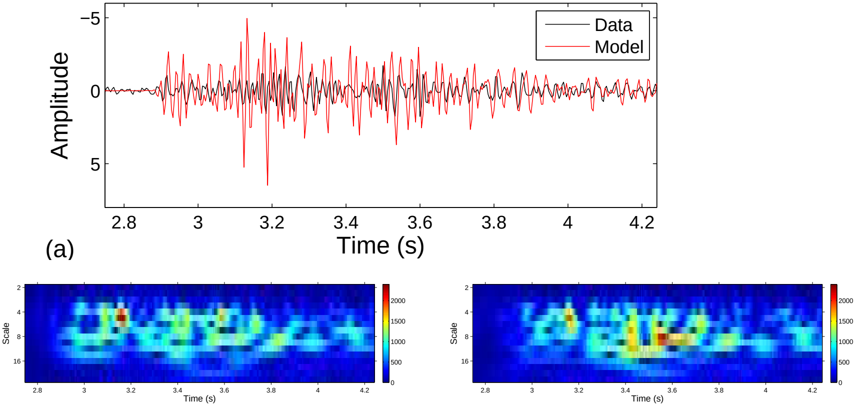

Figure 14: Which morphing is easier: time or time-scale?

Continuous wavelets

- Inversion with another wavelet ϕ

$$s(t) = \iint C_s(u,a)\phi_{u,a}(t)\frac{duda}{a^2}$$

$\Rightarrow$ time-scale domain processing! (back to the signal)

- Scalogram

$$|C_s(t,a)|^2$$

- Energy conversation

$$E = \iint |C_s(t,a)|^2 \frac{dtda}{a^2}$$

- Parseval-like formula

$$\langle s_1, s_2 \rangle = \iint C_{s_1}(t,a) C^*_{s_2}(t,a) \frac{dtda}{a^2}$$

Continuous wavelets

- Wavelet existence: admissibility criterion

$$0 < A_h = \int_0^{+\infty} \frac{\widehat{\Phi}^*(\nu)\Psi(\nu)}{\nu} d\nu = \int_{-\infty}^{0} \frac{\widehat{\Phi}^*(\nu)\Psi(\nu)}{\nu} d\nu < \infty$$

  generally normalized to 1
- Easy to satisfy (common freq. support midway 0 & ∞)
- With $\psi = \phi$, induces band-pass property:
  - necessary condition: $|\Phi(0)| = 0$, or zero-average shape
  - amplitude spectrum neglectable w.r.t. $|\nu|$ at infinity
- Example: Morlet-Gabor (non. adm.)

$$\psi(t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{t^2}{2\sigma^2}} e^{-\imath 2\pi f_0 t}$$

Discretization, redundancy and unary filters

Being practical again: deal with discrete signals

- Can one sample in time-scale (CWT):

$$C_s(\tau, a) = \int s(t)\psi^*_{\tau,a}(t)dt, \quad \psi_{\tau,a}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-\tau}{a}\right)$$

  with $c_{j,k} = C_s(kb_0a_0^j, a_0^j),\ (j,k) \in \mathbb{Z}$ and still be able to recover $s(t)$?
- Result 1 (Daubechies, 1984): there exists a wavelet frame if $a_0b_0 < C$, (depending on ψ). A frame is generally redundant
- Result 2 (Meyer, 1985): there exist an orthonormal basis for a specific ψ (non trivial, Meyer wavelet) and $a_0 = 2\ b_0 = 1$

Now: how to choose the practical level of redundancy?

Discretization

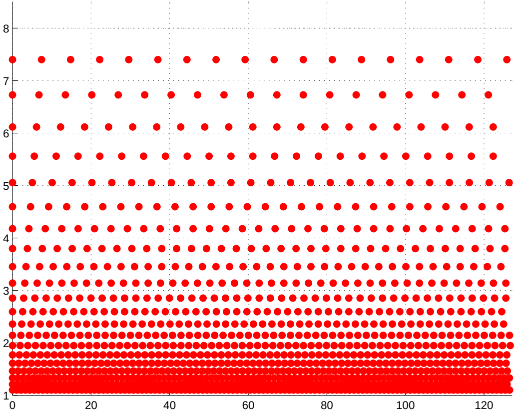

Figure 15: Wavelet frame sampling: $J = 21$, $b_0 = 1$, $a_0 = 1.1$

Discretization

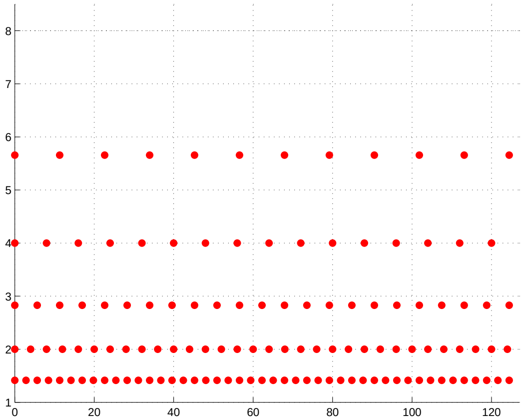

Figure 15: Wavelet frame sampling: $J = 5$, $b_0 = 2$, $a_0 = \sqrt{2}$

Discretization

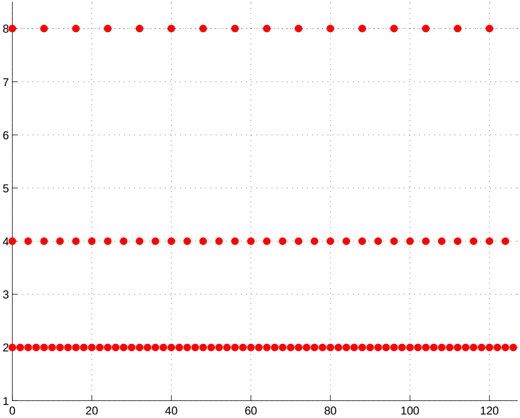

Figure 15: Wavelet frame sampling: $J = 3$, $b_0 = 1$, $a_0 = 2$

Discretization, redundancy and unary filters

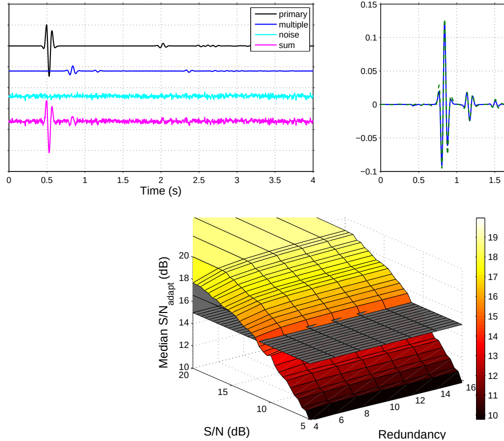

Figure 16: Redundancy selection with variable noise experiments

Discretization, redundancy and unary filters

- Complex Morlet wavelet:

$$\psi(t) = \pi^{-1/4} e^{-i\omega_0 t} e^{-t^2/2},\ \omega_0\text{: central frequency}$$

- Discretized time r, octave j, voice v:

$$\psi^v_{r,j}[n] = \frac{1}{\sqrt{2^{j+v/V}}} \psi\left(\frac{nT - r2^j b_0}{2^{j+v/V}}\right),\ b_0\text{: sampling at scale zero}$$

- Time-scale analysis:

$$\mathbf{d} = d^v_{r,j} = \left\langle d[n], \psi^v_{r,j}[n] \right\rangle = \sum_n d[n] \overline{\psi^v_{r,j}[n]}$$

Discretization, redundancy and unary filters

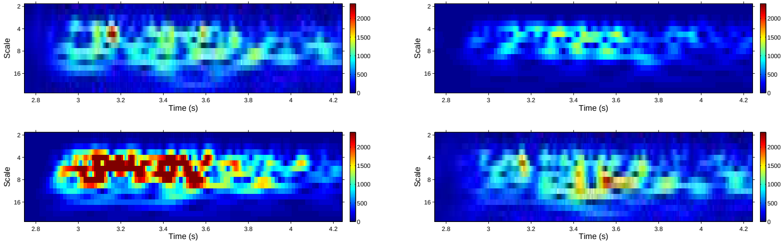

Figure 17: Morlet wavelet scalograms, data and models

Take advantage from the closest similarity/dissimilarity:

- remember the wiggle: on sliding windows, at each scale, a single complex coefficient compensates amplitude and phase

Discretization, redundancy and unary filters

- Windowed adaptation: complex $\mathbf{a}_{\text{opt}}$ compensates local delay/amplitude mismatches:

$$\mathbf{a}_{\text{opt}} = \underset{\{a_k\}(k \in K)}{\arg\min} \left\| \mathbf{d} - \sum_k a_k \mathbf{x}_k \right\|^2$$

- Vector Wiener equations for complex signals:

$$\langle \mathbf{d}, \mathbf{x}_m \rangle = \sum_k a_k \langle \mathbf{x}_k, \mathbf{x}_m \rangle$$

- Time-scale synthesis:

$$\hat{d}[n] = \sum_r \sum_{j,v} \hat{d}^v_{r,j} \widetilde{\psi}^v_{r,j}[n]$$

Results

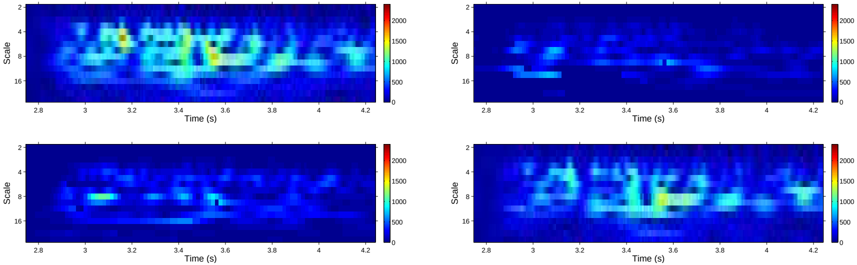

Figure 18: Wavelet scalograms, data and models, after unary adaptation

Results (reminders)

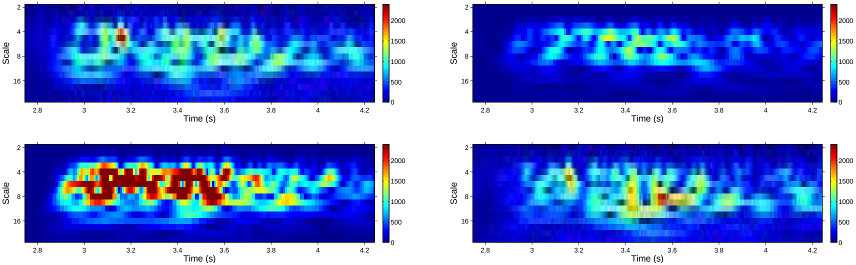

Figure 19: Wavelet scalograms, data and models

Results

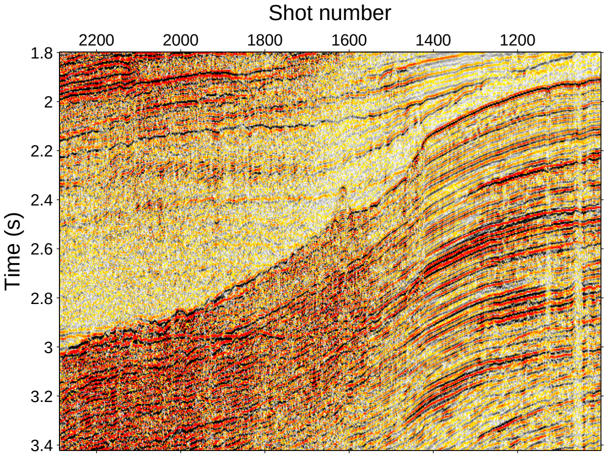

Figure 20: Original data

Results

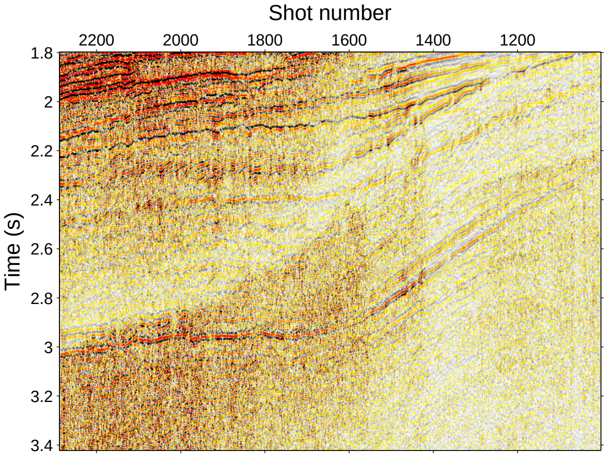

Figure 21: Filtered data, “best” model

Results

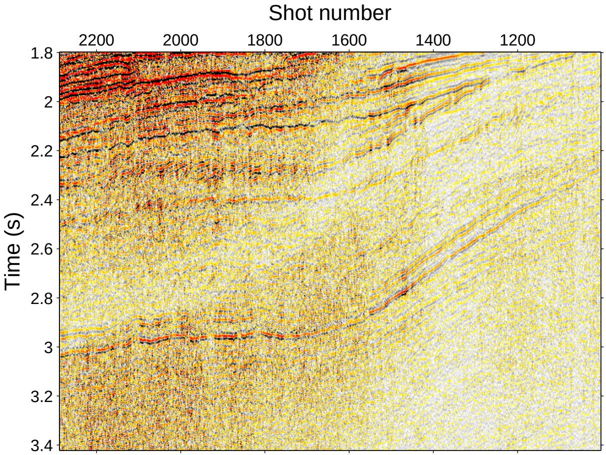

Figure 22: Filtered data, three models

Conclusions

Take-away messages:

- Technical side
  - Take good care of cascaded processing
  - Non-stationary, wavelet-based, adaptive multiple filtering
  - "Complex" wavelet transform + simple one-tap (unary) filter
  - Redundancy selection: noise robustness and processing speed
  - Smooth adaptation to adaptive joint multiple model filtering
- Practical side
  - Industrial integration
  - Competitive with more standard processing
  - Alternative results: less sensitive to random noises
- Future work: better integrate incoherent noise

Acknowledgements & references

Ventosa, S., S. Le Roy, I. Huard, A. Pica, H. Rabeson, P. Ricarte, and L. Duval, 2012, Adaptive multiple subtraction with wavelet-based complex unary Wiener filters: Geophysics, **77**, V183–V192; http://arxiv.org/abs/1108.4674

Jacques, L., L. Duval, C. Chaux, and G. Peyré, 2011, A panorama on multiscale geometric representations, intertwining spatial, directional and frequency selectivity: Signal Process., **91**, 2699–2730; http://arxiv.org/abs/1101.5320