Adaptive filtering in the complex wavelet domain with unary filters: application to multiple suppression in geophysics

On echoes and morphing

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On echoes and morphing

Figure 1: ... and adaptive subtraction
**Agenda**

1. Issues in geophysical signal processing
2. Problem: multiple reflections (*echoes*)
   - adaptive filtering with approximate models
3. Complex, continuous wavelets
   - and how they (may) simplify adaptive filtering
4. Discretization, redundancy and unary filters (*morphing*)
   - being practical: back to the discrete world
5. Results
6. Conclusion
Issues in geophysical signal processing

Figure 2: Seismic data acquisition and wave fields.
Issues in geophysical signal processing

Figure 3: Seismic data: aspect & dimensions (time, offset)
Issues in geophysical signal processing

Reflection seismology:

- seismic waves propagate through the subsurface medium
- seismic traces: seismic wave fields recorded at the surface
  - primary reflections: geological interfaces
  - many types of distortions/disturbances
- processing goal: extract relevant information for seismic data
- led to important signal processing tools:
  - $\ell_1$-promoted deconvolution (Claerbout, 1973)
  - wavelets (Morlet, 1975)
- exabytes ($10^6$ gigabytes) of incoming data
  - need for fast, scalable algorithms
Multiple reflections and models

Figure 4: Seismic data acquisition: focus on multiple reflections
Multiple reflections and models

Multiple reflections:

- seismic waves bouncing between layers
- one of the most severe types of interferences
- obscure deep reflection layers
- high cross-correlation between primaries \((p)\) and multiples \((m)\)
- additional incoherent noise \((n)\)
- \(d(t) = p(t) + m(t) + n(t)\)
  - model-based multiple attenuation: \(x_1(t), x_2(t), x_3(t)\)
- how to use approximate models?
Multiple reflections and models

Figure 5: Multiple reflections: data trace $d$ and model $x_1$
Multiple reflections and models

Multiple filtering:

- multiple prediction (correlation, wave equation) has limitations
- models are not accurate
  - \( m(t) \approx a_k(t)x_k(t - \tau_k(t)) \)?
  - standard: identify, apply a matching filter, subtract
- primaries and multiples are not (fully) uncorrelated
  - same (seismic) source
  - similarities/dissimilarities in time
  - similarities/dissimilarities in frequency
- variations in amplitude, waveform, delay
- issues in matching filter length:
  - short filters and windows: local details
  - long filters and windows: large scale effects
Multiple reflections and models

![Multiple reflections: data trace, model and adaptation](image)

**Figure 6:** Multiple reflections: data trace, model and adaptation
Multiple reflections and models

Figure 7: Multiple reflections: data trace and models, 2D version
Multiple reflections and models

- A long history of multiple filtering methods
  - general idea: combine adaptive filtering and transforms
    - data transforms: Fourier, Radon
    - enhance the differences between primaries, multiples and noise
    - reinforce the adaptive filtering capacity
  - intrication with adaptive filtering?
    - might be complicated (think about inverse transform)
- Main idea here:
  - exploit the non-stationary in the data
  - naturally allow both large scale & local detail matching
  - work in a complex domain: amplitude and phase representation
  - emulate an analytic signal representation (Hilbert transform)

⇒ Complex, continuous wavelets
- intermediate complexity in the transform
- hyper-simplicity in the (unary) adaptive filtering
Continuous wavelets

Figure 8: Complex wavelets at two different scales - 1
Continuous wavelets

Figure 9: Complex wavelets at two different scales - 2
Continuous wavelets

- Transformation group:
  \[ \text{affine} = \text{translation} (\tau) + \text{dilation} (a) \]

- Basis functions:
  \[
  \psi_{\tau,a}(t) = \frac{1}{\sqrt{a}} \psi \left( \frac{t - \tau}{a} \right)
  \]
  
  - \( a > 1 \): dilation
  - \( a < 1 \): contraction
  - \( 1/\sqrt{a} \): energy normalization
  - multiresolution (vs monoresolution in STFT)

\[
\psi_{\tau,a}(t) \xrightarrow{\text{FT}} \sqrt{a} \Psi(a f) e^{-i2\pi f \tau}
\]
Continuous wavelets

- **Definition**

\[ C_s(\tau, a) = \int s(t) \psi_{\tau,a}^*(t) dt \]

- **Vector interpretation**

\[ C_s(\tau, a) = \langle s(t), \psi_{\tau,a}(t) \rangle \]

projection onto time-scale atoms (vs time-frequency)

- **Redundant transform:** \( \tau \rightarrow \tau \times a \) “samples”

- **Parseval-like formula**

\[ C_s(\tau, a) = \langle X(f), \Psi_{\tau,a}(f) \rangle \]

\( \Rightarrow \) time-scale domain operations! (cf. Fourier)
Continuous wavelets

Introductory example

Figure 10: Noisy chirp mixture in time-scale & sampling
Continuous wavelets

Noise spread & feature simplification

Figure 11: Noisy chirp mixture in time-scale: scale, zoomed wiggle
Continuous wavelets

Figure 12: Which morphing is easier: time or time-scale?
Continuous wavelets

- Inversion with another wavelet $\phi$

\[ s(t) = \int \int C_s(u, a) \phi_{u,a}(t) \frac{dud\alpha}{\alpha^2} \]

$\Rightarrow$ time-scale domain processing! (back to the signal)

- Scalogram

\[ |C_s(t, \alpha)|^2 \]

- Energy conversation

\[ E = \int \int |C_s(t, \alpha)|^2 \frac{dtd\alpha}{\alpha^2} \]

- Parseval-like formula

\[ \langle s_1, s_2 \rangle = \int \int C_{s_1}(t, \alpha) C^*_{s_2}(t, \alpha) \frac{dtd\alpha}{\alpha^2} \]
Continuous wavelets

- Wavelet existence: admissibility criterion
  \[ 0 < A_h = \int_0^\infty \frac{\hat{\Phi}^*(\nu)\Psi(C')}{\nu} d\nu = \int_{-\infty}^0 \frac{\hat{\Phi}^*(\nu)\Psi(\nu)}{\nu} d\nu < \infty \]
  generally normalized to 1
- Induces band-pass property:
  - necessary condition: \(|\Phi(0)| = 0\), or zero-average shape
  - amplitude spectrum neglectable w.r.t. \(|\nu|\) at infinity
- examples: Morlet-Gabor (non. adm.)
  \[ \psi(t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{t^2}{2\sigma^2}} e^{-i2\pi f_0 t} \]
Multiple filtering
Continuous wavelets
Discret., redundancy, unary filters
Results & conclusion

Discretization, redundancy and unary filters

Being practical again: deal with discrete signals

- Can one sample in time-scale (CWT):

\[ C_s(\tau, a) = \int s(t) \psi^*_{\tau,a}(t) dt, \quad \psi_{\tau,a}(t) = \frac{1}{\sqrt{a}} \psi \left( \frac{t - \tau}{a} \right) \]

with \( c_{j,k} = C_s(kb_0a_0^j, a_0^j), \ (j, k) \in \mathbb{Z} \) and still be able to recover \( s(t) \)?

- Result 1 (Daubechies, 1984): there exists a wavelet frame if \( a_0b_0 < C \), (depending on \( \psi \)). A frame is generally redundant

- Result 2 (Meyer, 1985): there exist an orthonormal basis for a specific \( \psi \) (non trivial, Meyer wavelet) and \( a_0 = 2 \ b_0 = 1 \)

Now: how to choose the practical level of redundancy?
Discretization, redundancy and unary filters

Figure 13: Redundancy selection with variable noise experiments
Discretization, redundancy and unary filters

- Complex Morlet wavelet:
  \[ \psi(t) = \pi^{-1/4}e^{-i\omega_0 t}e^{-t^2/2}, \omega_0: \text{central frequency} \]

- Discretized time \( r \), octave \( j \), voice \( v \):
  \[ \psi_{r,j}^v[n] = \frac{1}{\sqrt{2^{j+v/V}}} \psi\left(\frac{nT - r2^jb_0}{2^j+v/V}\right), \ b_0: \text{sampling at scale zero} \]

- Time-scale analysis:
  \[ d = d_{r,j}^v = \langle d[n], \psi_{r,j}^v[n] \rangle = \sum_{n} d[n]\overline{\psi_{r,j}^v[n]} \]
Discretization, redundancy and unary filters

Figure 14: Morlet wavelet scalograms, data and models

Take advantage from the closest similarity/dissimilarity:

- remember the wiggle: on sliding windows, at each scale, a single complex coefficient compensates amplitude and phase
Discretization, redundancy and unary filters

- Windowed adaptation: complex $a_{opt}$ compensates local delay/amplitude mismatches:

$$a_{opt} = \text{arg min}_{\{a_k\}(k \in K)} \left\| d - \sum_k a_k x_k \right\|^2$$

- Vector Wiener equations for complex signals:

$$\langle d, x_m \rangle = \sum_k a_k \langle x_k, x_m \rangle$$

- Time-scale synthesis:

$$\hat{d}[n] = \sum_r \sum_{j, v} \hat{d}_r^v \bar{\psi}_{r, j}[n]$$
Results

Figure 15: Wavelet scalograms, data and models, after unary adaptation
Results (reminders)

Figure 16: Wavelet scalograms, data and models
Results

Figure 17: Original data
Results

Figure 18: Filtered data
Conclusions

Take-away messages:

- **Technical side**
  - Take good care of cascaded processing
  - Non-stationary, wavelet-based, adaptive multiple filtering
  - “Complex” wavelet transform + simple one-tap (unary) filter
  - Redundancy selection: noise robustness and processing speed
  - Smooth adaptation to adaptive joint multiple model filtering

- **Practical side**
  - Industrial integration
  - Competitive with more standard processing
  - Alternative results: less sensitive to random noises

- **Future work**: better integrate incoherent noise
Acknowledgements & references
