LAPPED TRANSFORM DOMAIN DENOISING USING HIDDEN MARKOV TREES

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ABSTRACT

Algorithms based on wavelet-domain hidden Markov tree (HMT) have demonstrated excellent performance for image denoising. The HMT model is able to capture image features across the scales, in contrast to classical shrinkage that thresholds subbands independently.

In this work, we extend the aforementioned results to a lapped transform domain. Lapped Transforms (LT) are M-channel linear phase filter banks. Their use is motivated by their good energy compaction properties and robustness to oversmoothing. It is also observed that LT preserve better oscillatory image components, such as textures.

Since LT are applied as block transforms, the transforms coefficients are rearranged into an octave-like decomposition, and their statistics are modeled by the same HMT structure as in the wavelet case. At moderate noise levels, the proposed algorithm is able to improve the results obtained with wavelets, subjectively and objectively.

1. INTRODUCTION AND MOTIVATIONS

The discrete wavelet transform (DWT) provides sparse representations for images. As a consequence, numerous DWT-based algorithms have been proposed in the past years for efficient signal and image statistical analysis. For instance, wavelet-domain thresholding provides asymptotically optimal performance in the case of Gaussian additive noise [1]. The key to noise filtering is to transform the signal and the noise to a domain where their statistics are modeled more efficiently, via appropriate orthogonal transforms. Moreover, wavelet decompositions exhibit two heuristic properties often termed ”clustering” and ”persistence”: feature-related wavelet coefficients (edges or singularities) tend to cluster locally in a subband and to persist across scales, through the wavelet tree. Recently, algorithms adopted tree-adapted subband-dependent shrinkage [2, 3]. Also, sophisticated models of the joint statistics may be useful for capturing key-features in real-world images. A recent approach relies on Markov random fields. We refer to [4, 5] for an rich overview of their use in signal and image processing. Based on the Hidden Markov Tree framework developed in [5], H. Choi *et al.* have proposed efficient image denoising [6] as well as robust SAR segmentation [7].

The proposed work extends the use of hidden Markov models to a lapped transform (LT) domain. LT are usually viewed as block-transforms. Though, T. Tran *et al.* [8] have demonstrated that well-designed LT are able to improve on DWT for natural image compression, in the Embedded Zerotree [9] framework. In the context of denoising, the LT coefficients are rearranged into an octave-like representation. The resulting ”scales” bear the same clustering and persistence properties as in the wavelet representation. Moreover, LT design may enforce both orthogonality and linear-phase (in contrast to non-Haar 1D wavelets), as well as attractive additional degrees of freedom in design. In the following, we first briefly review some properties of the Lapped Transforms. We then describe the dyadic re-mapping of the transformed coefficients, and basic principles behind Hidden Markov Tree models. The proposed algorithm is then applied to natural image denoising. We conclude by comments on foreseen improvements of the present work.

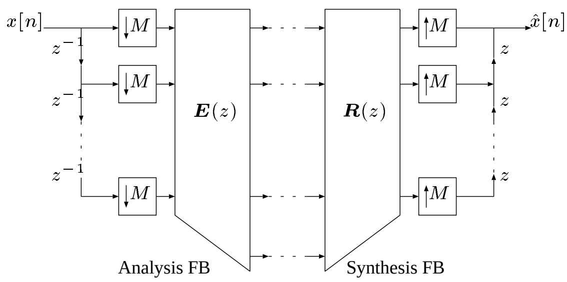


Fig. 1. Block diagram of the polyphase matrices of a processing system based on a M-band critically sampled filter bank.

2. LAPPED TRANSFORMS

The Lapped Orthogonal Transform (LOT, [10]) as been developed to overcome the annoying blocking effects of non overlapping block transforms such as the DCT. More generally, Lapped transforms are defined as linear phase paraunitary filter banks (FB). A block diagram of the analysis and synthesis FB pair is given in Figure 1. The analysis and the synthesis M-band FB polyphase matrices ($\mathbf{E}(z)$ and $\mathbf{R}(z)$ respectively) provide perfect reconstruction with zero delay if and only if:

$$\mathbf{R}(z)\mathbf{E}(z) = \mathbf{I}_M,$$

where $\mathbf{I}_M$ is the identity matrix [11, p. 304 sq.]. LT may be parameterized through efficient lattice structures for cost-driven optimization. We refer to [10, 11, 8] for a comprehensive overview on Lapped Transforms.

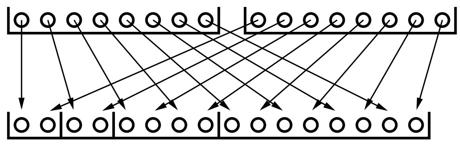


Fig. 2. Dyadic rearrangement of 1D LT coefficients: (Top) block-transform with uniform frequency partition, (Bottom) octave-like representation.

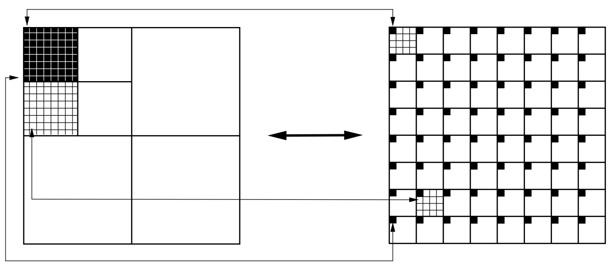


Fig. 3. Dyadic equivalence between 2D wavelet and LT coefficients: (Left) two-level octave-like representation, (Right) four-channel block-transform with uniform frequency partition.

LT project signals onto M equally spaced frequency bands, in contrast to the octave-band wavelet representation. When M is a power of 2 (typically 8 or 16), the transformed coefficients bear an octave-like grouping, with $J = \log_2 M$ decomposition levels. For one group of M transformed coefficients, the DC component is assigned to the lower scale subband. Then, from low to high frequencies, the kth subband is formed respectively from the next group of $2^k/2$ coefficients. The $J+1$ groups are then associated with respect to the block position in the signal. Figure 2 represents the dyadic rearrangement from two consecutive blocks. Two blocks of $8 = 2^3$ coefficients (dots on top) into $J = 4$ groups yield a three-level decomposition (dots on bottom of Fig. 2).

The re-mapping from a four channel block transform to a two level dyadic transform is depicted in Fig. 3. The right-hand side image is made of 8×8 subblocks. Each subblock gathers 4×4 coefficients (see the top left subblock), where the black squares represent the DC component. In a fashion similar to the 1D case, all the 64 DC coefficients are grouped into a 8×8 group (top left of the left-hand side image) representing the low-pass component of the dyadic representation. Arrows between coefficients link reciprocal locations of coefficients in the dyadic and the block grouping scheme. Once wavelet and LT coefficients share similar grouping, the same denoising procedure may be applied to both domains.

3. TRANSFORM-BASED DENOISING BASED ON HIDDEN MARKOV TREE MODEL

Under the additive noise assumption, an image and its noisy observation is usually modeled as

$$y(i,j) = x(i,j) + n(i,j),$$

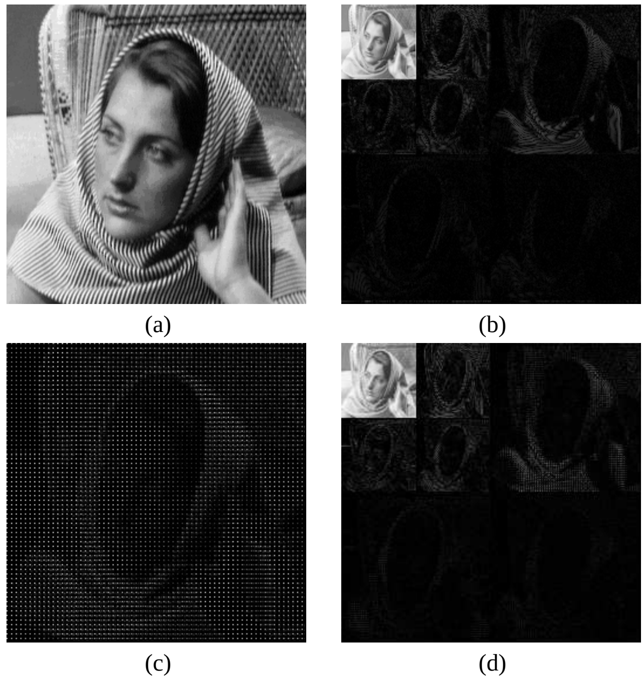


Fig. 4. (a) Original Barbara image, (b) Wavelet-db8 decomposition, (c) four-channel DCT block decomposition, (d) dyadic remapping from image (c).

where n is a zero-mean Gaussian noise with known variance σ^2. Since we have chosen orthogonal transforms, n keeps the same properties in the transformed domain. The joint probability density function of the family of images that x belongs to is often unattainable. Based on wavelet approximate decorrelation, simpler models have been proposed for coefficient modeling. The simplest independent Gaussian models generally obtain improvements from residual inter-coefficients dependencies.

The Hidden Markov Tree (HMT) model is often described as a quad-tree structured probabilistic graph that captures the statistical properties of the wavelet transforms of images. The marginal *pdf* is modeled as a Gaussian mixture with two components. The hidden states refer to the large or small nature of the coefficient.

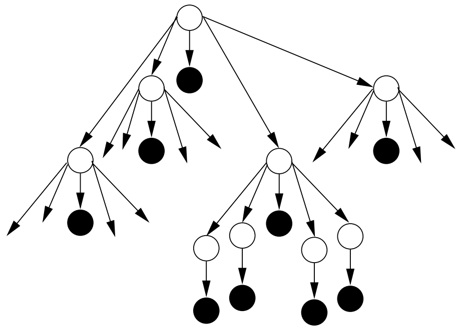


Fig. 5. Diagram of a Hidden Markov Tree in a quad-tree. White dots represent hidden states with arrows as dependencies, black dots the wavelet coefficients.

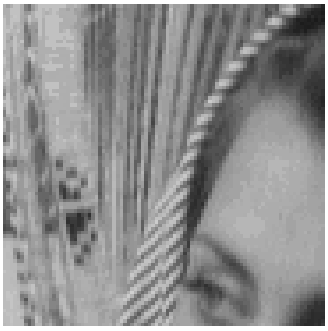


Fig. 6. A segment of the Barbara image, original detail.

Since the coefficient nature tends to propagate across scales (see [9, 6]), the Hidden Markov Tree materializes the cross-scale link between the hidden states. A template HMT is depicted in Fig. 5.

The parameters of the HMT model are trained for a set of images using an Expectation Maximization algorithm. We refer to [5, 6] for details on the implementation of Hidden Markov Trees.

σ	Noisy (PSNR)	db8-HMT	LOT-HMT
7.6	30.46	32.82	33.38
12.7	25.98	29.74	30.15
17.8	23.09	27.82	28.21
20.4	21.89	27.01	27.19
22.9	20.92	26.37	26.43
24.2	20.43	26.09	26.16
25.5	19.97	25.93	25.87
33.1	17.73	24.49	24.23

Table 1. HMT denoising results with the Barbara image at several noise levels (σ is the noise standard deviation, PSNR in dB).

4. APPLICATION TO IMAGE DENOISING

The experiments of this work have been performed on the Barbara image. It is relatively rich in textures that are often oversmoothed with classical wavelet shrinkage method.

Octave and block-domain representations of the Barbara image are given in Fig. 4. Figure 4b results from a two-level wavelet decomposition of Fig. 4a with the 8-tap orthogonal Daubechies wavelet (db8). Though the high-pass coefficients are low in magnitude (dark tones), edges and texture related coefficients cluster and propagate across scales, as observed for efficient compression in [9]. Fig. 4c represents the block transformed image using a four-channel 8-tap coding gain optimized Lapped Orthogonal Transform. The brighter pixels are mainly DC components of each 4×4 subblock. The coefficients are rearranged to an octave-like domain in Fig. 4d, according to the procedure described in Section 2. For more clarity, the constrast have been enhanced by displaying the square root of the coefficient magnitudes. It results in increased brightness for low magnitude cofficients. As expected, similar clustering and persistence of coefficients are observed after octave remapping.

In the following, we compare results obtained from the db8 wavelet and a coding gain optimized 8-channel 16-tap Lapped Orthogonal Transform. For fair comparison between the two transforms, the DWT has three levels of decomposition, which gives equivalent depth to the $8 = 2^3$ channels of the Lapped Transform.

(a) Noisy image (26 dB)

(b) db8-HMT (29.74 dB)

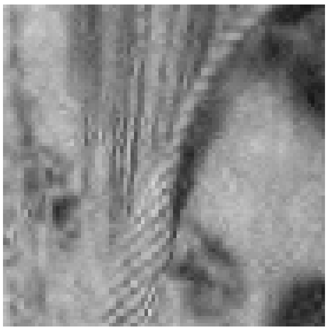
(c) LOT-HMT (30.15 dB)

Fig. 7. Barbara image (a) Noisy image at 26 dB, (b) Wavelet result, (c) LOT result.

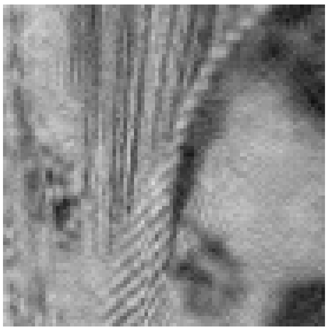
Table 1 compares objective results for HMT denoising. Both transforms yield good denoising performance for the Barbara image, with up to 6 dB improvement on the noisy image at 17-20 dB noise levels. LT-HMT denoising outperforms the wavelet for noise levels above 20 dB. For higher noise levels, the LOT performance decreases in PSNR.

Figures 7 and 8 display a detail from Barbara (Fig. 8) at 25.98 dB and 19.97 dB PSNR noise level respectively. Figure 7 demonstrates that even with a weak PSNR improvement (0.4 dB), edges and textures are better preserved with the LOT than with the wavelet. It can be seen from the diagonal stripes of the scarf on the bottom center of the picture. Vertical details on the background wicker chair are also slightly oversmoothed with the wavelet transform.

(a) Noisy image (20.43 dB)



(b) db8-HMT (26.09 dB)



(c) LOT-HMT (26.16 dB)

Fig. 8. A segment of the Barbara image at 20.43 dB (a) Noisy image, (b) Wavelet result, (c) LOT result.

In Figure 8, the PSNR is slightly higher after wavelet-based HMT denoising. Textures are nevertheless better preserved with LT-based denoising, as wavelet oversmoothing clearly appear.

5. CONCLUSIONS

We propose a Hidden Markov Tree based denoising algorithm in the Lapped Transform domain. It relies on a octave-like re-mapping of the LT coefficients. In this scheme, a 8-channel 16-tap LOT is able to outperform wavelets in PSNR for moderate noise level. At higher noise levels, it preserves textures and edges better. The proposed method inherits from some of the LOT attractive properties (orthogonality, linear phase and robustness to oversmoothing) combined with the effectiness of the Hiddden Markov Tree modeling of the image features across scales.

Future developments will involve an increase of the decomposition level by applying a wavelet transform to the lower octave band of both the wavelet and the LT decomposition, as well as the design of LT to increase the sparse nature of the decomposition.

6. ACKNOWLEDGMENTS

The first author would like to thank Hatem Elloumi for programming some of the filter banks routines used in this work. We also thank the reviewers for their constructive remarks and for pointing out missing details.

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